YEAR 10
KNOWLEDGE ORGANISERS

White Rose Maths

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ALGEBRA...

Algebraic manipulation

What do I need to be able to do?

- Substitution
- Collect like terms
- Simplify expressions
- Addition and subtraction laws for indices
- Powers of powers
- Expand a single bracket
- Factorise into a single bracket

Keywords

Tier 2:

Variable: a symbol for a number we don't know yet.

Expression: numbers, symbols and operators grouped together to show the value of something

Identity: An equation where both sides have variables that cause the same answer includes ≡

Substitute: replace one variable with a number or new variable.

sign)

Equivalent: something of equal value

Coefficient: a number used to multiply a variable

Tier 3:

Highest Common Factor (HCF): the biggest factor (or number that multiplies to give a term)

Using letters to represent numbers

5 + 5 + 5	y + y + y + y	20 ÷ h
3 x 5	y x 4	$\frac{20}{h}$
5 x 3	4 x y	
	4y	

Addition and multiplication can be done in any order

Commutative calculations

4 lots of 'y'

20 shared into 'h' number of groups

Substitution into expressions

4y ← 4 lots of 'y'

If y = 7 this means the expression is asking for 4 'lots of' 7

4 x 7 **OR** 7 + 7 + 7 + 7 **OR** 7 x 4 $\boxed{= 28}$

eg: y − 2

= 7 − 2 = 5

$\boxed{2(x + 3)}$

Put the expression into a function machine

INPUT → + 3 → x 2 → OUTPUT

If x = 10   10 + 3 = 13…. 13 x 2 = 26

Like and unlike terms

Like terms are those whose variables are he same

♡   3♡   are **like** terms

the variable is the same

☆   3♡   are **unlike** terms

the variables are NOT the same

Examples and non-examples

Like terms	Un-like terms
y, 7y	y, 7x
$2x^2$, x^2	$2x^2$, $2c^2$
ab, 10ba	ab, 10a
5, -2	5, -2t

Note here ab and ba are commutative operations, so are still like terms

Collecting like terms ≡ symbol

The ≡ symbol means equivalent to.

It is used to identify equivalent expressions

Collecting like terms

Only **like terms** can be combined

4x +5b −2x +10b

4x +5b −2x +10b

2x + 15b

Common misconceptions

$2x + 3x^2 + 4x \equiv 6x + 3x^2$

Although they both have the x variable x2 and x terms are un-like terms so can not be collected

Algebraic constructs

Expression

A sentence with a minimum of two numbers and one maths operation

Term

A single number or variable

Identity

An equation where both sides have variables that cause the same answer includes ≡

Multiply single brackets

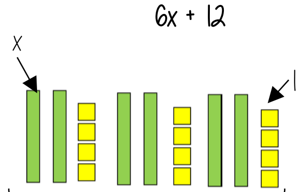
3 (2x + 4)

	2x	4
3	3 x 2x 6x	3 x 4 12

6x + 12

Different representations of

3(2x+4) = 6x + 12



6x + 12

2x + 4			2x + 4			2x + 4		
x	x	4	x	x	4	x	x	4
6x + 12								

Factorise into a single bracket

8x + 4	4
2x + 1	

Try and make this the **highest common factor**

The two values **multiply** together (also the area) of the rectangle

8x + 4 ≡ 4 (2x + 1)

Note:

8x + 4 ≡ 2(4x + 2)

This is factorised but the HCF has not been used

Evaluate algebraic expressions

a = 5 b = -4

$a^2 = 5^2$
$a^2 = 25$

$b^2 = (-4)^2$
$b^2 = 16$

With negative numbers the brackets are important so that it performs -4 x -4.

Brackets around negative substitutions helps remove calculation errors

$2a - b = 2 \times 5 - (-4) = 10 + 4 = \underline{14}$

$3b - 2a = 3(-4) - 2(5) = -12 - 10 = \underline{-22}$

Higher powers and roots

x^n — n – power (number of times multiplied by itself); x – the base number.

$\sqrt[n]{x}$ — Finding the nth root of any value

Other mental strategies for square roots

$\sqrt{810000}$

$= \sqrt{81} \times \sqrt{10000}$

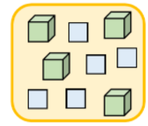
$= 9 \times 100$

$= 900$

Addition/ Subtraction with indices

Coefficient Power

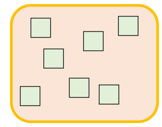
$\underbrace{\underbrace{5x^2}_{\text{Term}} + \underbrace{4x^4}_{\text{Term}}}_{\text{Expression}}$ →



Each square represents x^2 and each cube represents x^4

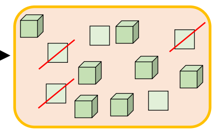
Only similar terms can be simplified
If they have different powers, they are unlike terms

$5x^2 + 2x^2$ →



→ $7x^2$

$5x^2 + 6x^4 - 3x^2 + x^4$ →



→ $2x^2 + 7x^4$

Multiply expressions with indices

$4b \times 3a$
$\equiv 4 \times b \times 3 \times a$
$\equiv 4 \times 3 \times b \times a$
$\equiv 12ab$

$5t \times 9t$
$\equiv 5 \times t \times 9 \times t$
$\equiv 5 \times 9 \times t \times t$
$\equiv 45t^2$

$2b^4 \times 3b^2$
$\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b$
$\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b$
$\equiv 6b^6$

There are often misconceptions with this calculation but break down the powers

Divide expressions with indices

$\frac{24}{36} \rightarrow \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 2 \times \cancel{3}} \rightarrow \frac{2}{3}$

$\frac{5a^3b^2}{15ab^6} \rightarrow \frac{\cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{\underbrace{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times b \times b}} \rightarrow \frac{a^2}{3b^4}$

Cross cancelling factors shows cancels the expression

$\frac{23a^7y^2}{5db^6}$ This expression cannot be divided (cancelled down) because there are no common factors or similar terms

Addition/ Subtraction laws for indices

$3^5 \times 3^2$
$= \underbrace{(3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)}$ → 3^7

The base number is all the same so the terms can be simplified

Addition law for indices

$a^m \times a^n = a^{m+n}$

$3^5 \div 3^2 \rightarrow 3^3$

$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \rightarrow \frac{3^3}{3^0} \rightarrow \frac{3^3}{1}$

Subtraction law for indices

$a^m \div a^n = a^{m-n}$

Powers of powers

$(x^a)^b = x^{ab}$

$(2^3)^4 = \underbrace{2^3 \times 2^3 \times 2^3 \times 2^3}$

The same base and power is repeated. Use the addition law for indices

$(2^3)^4 = 2^{12}$ $a \times b = 3 \times 4 = 12$

NOTICE the difference

$(2x^3)^4 = \underbrace{2x^3 \times 2x^3 \times 2x^3 \times 2x^3}$

$(2x^3)^4 = 16x^{12}$

The addition law applies ONLY to the powers. The integers still need to be multiplied

Fractional indices(H)

$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

The denominator of a fractional power acts as a 'root'.
The numerator of a fractional power acts as a normal power.

ALGEBRA... Equations, inequalities and formulae

What do I need to be able to do?

- Solve equations
- Solve fractional equations
- Solve equations with unknowns on both sides
- Understand inequalities
- Solve inequalities
- **Represent solutions to inequalities using set notation (E)**
- Change the subject of a known formula
- Change the subject of a simple formula
- Change the subject of a complex formula
- **Change the subject where the subject appears more than once (E)**

Keywords

Tier 2:

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet.

Equation: an equation says that two things are equal – it will have an equals sign =

Expression: numbers, symbols and operators grouped together to show the value of something

Identity: An equation where both sides have variables that cause the same answer includes ≡

Linear: an equation or function that is the equation of a straight line

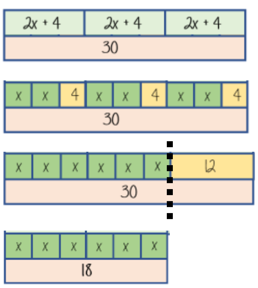
Intersection: the point that two lines meet

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.

Formula: a fact or rule that uses mathematical symbols

Solve equations

$3(2x + 4) = 30$



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

-12 -12

Substitute to check your answer. This could be negative or a fraction or decimal

$6x = 18$

$\div 6$ $\div 6$

x
3

$x = 3$

Form and solve inequalities

Two more than treble my number is greater than 11

Form

$x \rightarrow \times 3 \rightarrow +2 \rightarrow 11$

$3x + 2 > 11$

Solve

$x \leftarrow \div 3 \leftarrow -2 \leftarrow 11$

$x > 3$

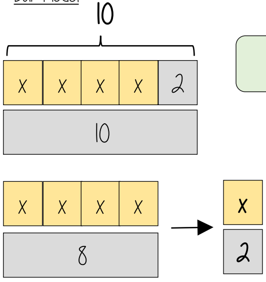
Two-step equations

$4x + 2 = 10$

$10 - 4x = 2$

Representing the same question (use fact families)

Bar Model



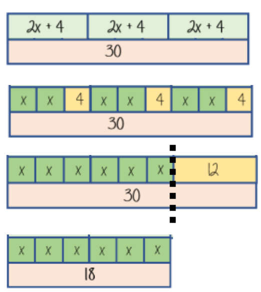
Function machine

$x \rightarrow \boxed{\times 4} \rightarrow \boxed{+2} \rightarrow 10$

Inverse operations to find x

Solve equations with brackets

$3(2x + 4) = 30$



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Substitute to check your answer. This could be negative or a fraction or decimal

$6x = 18$

$\div 6$ $\div 6$

x
3

$x = 3$

Simple Inequalities

$<$ less than

$>$ more than

$\leq$ less than or equal to

$\geq$ more than or equal to

$x + 2 \leq 20$

"my value + 2 is less than or equal to 20"

$x \leq 18$

The biggest the value can be is 18

Note:
$x<10$ and $10>x$
represent the same values

$x < 10$
Say this out loud
"x is a value less than 10"

$10 > x$
Say this out loud
"10 is more than the value"

Formulae and Equations

Formulae – all expressed in symbols

Equations – include numbers and can be solved

Substitute in values

Solutions on a number line

$x < 1$ $x \leq 1$ $x > 1$ $x \geq 1$

Both represent values less than 1

Includes the value 1

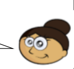
Both represent values more than 1

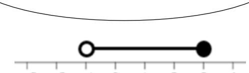
Includes the value 1

● Includes the value it sits above

○ Does NOT include the value it sits above

Values less than or equal to 3 but also more than -1



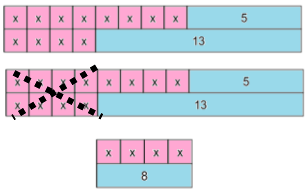


$-1 < x \leq 3$

This includes the integer values 0,1,2,3

Equations: unknown on both sides

$8x + 5 = 4x + 13$



$$\begin{aligned} 8x + 5 &= 4x + 13 \\ -4x \quad & \quad -4x \\ 4x + 5 &= 13 \\ -5 \quad & \quad -5 \\ 4x &= 8 \\ \div 4 \quad & \quad \div 4 \\ x &= 2 \end{aligned}$$

Inequalities: unknown on both sides

$8x + 5 \le 4x + 13 \longrightarrow x \le 2$

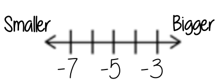
Only value 2 or less will satisfy this inequality

Inequalities with negatives

Method 1 Make x positive first

$$\begin{aligned} 2 - 3x &> 17 \\ +3x \quad & \quad +3x \\ 2 &> 17 + 3x \\ -17 \quad & \quad -17 \\ -15 &> 3x \\ \div 3 \quad & \quad \div 3 \\ \underline{-5} &\underline{> x} \end{aligned}$$

x is true for any value smaller than -5



✓ CHECK IT!
$2 - 3(-6) = 20$
TRUE/ CORRECT

Method 2 Keep the negative x

$$\begin{aligned} 2 - 3x &> 17 \\ -2 \quad & \quad -2 \\ -3x &> 15 \\ \div -3 \quad & \quad \div -3 \\ \underline{x} &\underline{> -5} \end{aligned}$$

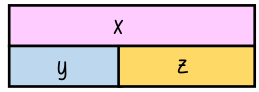
x is true for any value bigger than -5

This cannot be true…

$\underline{x < -5}$

When you multiply or divide x by a negative you need to reverse the inequality

Rearranging Formulae (one step)



$x = y + z$

Rearrange to make y the subject.

$\underline{y = x - z}$

$y \xrightarrow{+z} x$

$y \xleftarrow{-z} x$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging…

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$$\begin{aligned} 4x - 3 &= 9 \\ +3 \quad & \quad +3 \\ 4x &= 12 \\ \div 4 \quad & \quad \div 4 \\ \underline{x} &\underline{= 3} \end{aligned}$$

In a formula (make x the subject)

$$\begin{aligned} xy - s &= a \\ +s \quad & \quad +s \\ xy &= a + s \\ \div y \quad & \quad \div y \\ x &= \frac{a + s}{y} \end{aligned}$$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \frac{4x + 9}{2}$

Rearranging Complex Formulae(H)

$s = vt - \frac{1}{2}at^2$ is a formula used in maths and physics.

Express a in terms of s, v and t.

$$\begin{aligned} s &= vt - \tfrac{1}{2}at^2 && +\tfrac{1}{2}at^2 \\ s + \tfrac{1}{2}at^2 &= vt && -s \\ \tfrac{1}{2}at^2 &= vt - s && \div t^2 \\ \tfrac{1}{2}a &= \frac{vt - s}{t^2} && \times 2 \\ a &= \frac{2(vt - s)}{t^2} \end{aligned}$$

ALGEBRA… Quadratic expressions and equations

What do I need to be able to do?

- Expand double brackets
- Expand triple brackets
- Factorise quadratic expressions
- Factorise more complex quadratic expressions (E)
- Difference of two squares
- Solve quadratic equations equal to 0
- Solve quadratic equations by factorisation
- Solve more complex quadratic equations by factorisation (E)
- Complete the square
- Solve quadratic equations by completing the square (E)
- Complete the square with more complex quadratic expressions (E)
- Solve quadratic equations using the quadratic formula

Keywords

Tier 2:

Factorise: put in brackets by taking out the highest common factors

Coefficient: the number in front of the variable

Binomial: a polynomial with two terms

Quadratic: a polynomial with four terms (often simplified to three terms)

Tier 3:

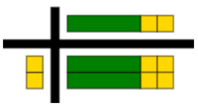
Difference of two squares: Two terms that are squared and separated by a subtraction sign like this: $a^2 - b^2$

Complete the square: solve a quadratic equation by changing the form of the equation so that the left side is a perfect square

Surd: used to write irrational numbers precisely

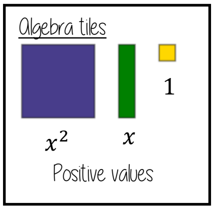
Expand double brackets

$2(x+2) \equiv 2x + 4$

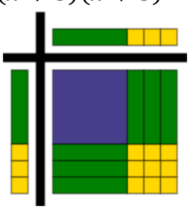


Algebra tiles can represent a binomial expansion

Has two terms



$(x+3)(x+3) \equiv x^2 + 6x + 9$



This is a quadratic. It has four terms which simplified to three terms

The order of the brackets has no impact on the outcome. e.g. $(x+3)(3+x)$

Factorising Quadratics

When a quadratic expression is in the form $x^2 + bx + c$ find the two numbers that **add to give b** and **multiply to give c**.

$x^2+7x+10 = (x+5)(x+2)$

(because 5 and 2 add to give 7 and multiply to give 10)

Difference of Two Squares

An expression of the form a^2-b^2 can be factorised to give $(a+b)(a-b)$

$x^2 - 25 = (x+5)(x-5)$

$16x^2 - 81 = (4x+9)(4x-9)$

Solving Quadratics

Factorise the quadratic in the usual way and then **solve** = 0

Solve $x^2 + 3x - 10 = 0$

Factorise: $(x+5)(x-2) = 0$

$x = -5$ or $x = 2$

Directed numbers

+ + → +

− − → +

+ − → −

− + → −

eg $a = -5$ and $b = 2$

$a^2 = a \times a = -5 \times -5 = 25$

$b + a = 2 + -5 = -3$

Factorising Quadratics when a ≠ 1 (H)

When a quadratic is in the form $ax^2 + bx + c$

1. Multiply a by c = ac
2. Find two numbers that add to give b and multiply to give ac.
3. Re-write the quadratic, replacing bx with the two numbers you found.
4. Factorise in pairs – you should get the same bracket twice
5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.

Factorise $6x^2 + 5x - 4$

1. $6 \times -4 = -24$
2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3
3. $6x^2 + 8x - 3x - 4$
4. Factorise in pairs: $2x(3x+4) - 1(3x+4)$
5. Answer = $(3x+4)(2x-1)$

Completing the Square (H)

A quadratic in the form $x^2 + bx + c$ can be written in the form $(x+p)^2 + q$

1. Write a set of brackets with x in and half the value of b.
2. Square the bracket.
3. Subtract $\left(\frac{b}{2}\right)^2$ and add c.
4. Simplify the expression.

Complete the square of $y = x^2 - 6x + 2$

Answer:

$(x-3)^2 - 3^2 + 2$

$= (x-3)^2 - 7$

a ≠ 1

A quadratic in the form $\mathbf{ax^2 + bx + c}$ can be written in the form $p(x+q)^2 + r$

Use the same method as above, but factorise out a at the start.

Complete the square of $4x^2 + 8x - 3$

Answer:

$4[x^2 + 2x] - 3$

$= 4[(x+1)^2 - 1^2] - 3$

$= 4(x+1)^2 - 4 - 3$

$= 4(x+1)^2 - 7$

Expanding triple brackets

Expand and simplify the following, $(x+1)(x+3)(x-2)$

Step 1 – Multiply out the first two brackets $(x+1)(x+3)$

$= x^2 + 3x + x + 3$

$= x^2 + 4x + 3$

Step 2 – Multiply the result $(x^2 + 4x + 3)$ by $(x-2)$

$(x^2 + 4x + 3)(x-2)$

$= x^3 + 4x^2 + 3x - 2x^2 - 8x - 6$

$= x^3 + 2x^2 - 5x - 6$

Quadratic Formula (H)

A quadratic in the form $ax^2 + bx + c = 0$ can be solved using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the formula if the quadratic does not factorise easily

Solve $3x^2 + x - 5 = 0$

Answer:

$a = 3, b = 1, c = -5$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 3 \times -5}}{2 \times 3}$$

$$x = \frac{-1 \pm \sqrt{61}}{6}$$

$x = 1.14$ or -1.47 (2 d.p.)

PROPORTION... Percentages

What do I need to be able to do?

- Percentage of an amount
- Percentage increase and decrease
- Repeated percentage change
- Express one number as a fraction or a percentage of another
- Express a change as a percentage
- Find the original value after a percentage change
- Simple interest
- Compound interest
- Choose appropriate methods to solve percentage problems

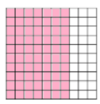
Keywords

Tier 2:
Compound interest: calculating interest on both the amount plus previous interest
Depreciation: a decrease in the value of something over time.
Growth: where a value increases in proportion to its current value such as doubling.
Decay: the process of reducing an amount by a consistent percentage rate over time.
Multiplier: the number you are multiplying by
Equivalent: of equal value.
Tier 3:
Exponent: how many times we use a number in multiplication. It is written as a power

Compare FDP

Comparisons are easier in the same format

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares, 70 "hundredths" = 7 "tenths" 0.7 →



→ 70 hundredths = 70%

Using a calculator

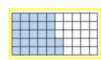
$\frac{\blacksquare}{\square}$ → S⇌D Convert to a decimal

This will give you the answer in the simplest form

× 100 converts to a percentage

Be careful of recurring decimals
eg $\frac{1}{3}$ = 0.3333333 = $0.\dot{3}$
The dot above the 3

Express as a percentage



27 per every 50 shaded $\frac{27}{50}$ → 54 per every 100 shaded $\frac{54}{100}$ → 54%

$\frac{13}{30}$ → $\frac{\blacksquare}{\square}$ → $\frac{13}{30}$ → × 100 → 43.3333....% 43%

Can't use equivalence easily to find 'per hundred' → Decimal percentages are still a percentage

Percentage increase/decrease

100% | 42% | Decrease by 58%

100% − 58% = 42%
100 − 0.58 = 0.42 ← Multiplier Less than 1

100% | 12% | Increase by 12%

100% + 12% = 112%
100 + 0.12 = 1.12 ← Multiplier More than 1

Percentage change

$$\frac{\text{Difference in values}}{\text{Original value}} \times 100$$

I bought a phone for £200. A year later sold it for £125.
Percentage loss

$200 \times \frac{75}{100} = 37.5\%$

I bought a house for £180,000, I later sold it for £216,000
Money made (profit value)

$\frac{36000}{180000} \times 100 = 20\%$

Simple and compound interest

Simple Interest

James invests £2000 at 5% simple interest

100% | 5%
£2000 | £100

The original value increases by this amount every year

Compound Interest

Tess invests £100 at 10% compound interest for 3 years

£100 Original amount: £100
£100 | 10% £10 Y1: £110
£110 | 10% £11 Y2: £121
£121 | 10% £12.1 Y3: £132.10

The multiplier 1.10 repeats each year

Repeated percentage change

Compound Interest

Tess invests £100 at 10% compound interest for 3 years

£100 x 1.10 x 1.10 x 1.10

$£100 \times 1.10^{3}$

Original amount | Repeated multiplier | Number of occurrences

Depreciation

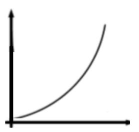
Depreciation calculations use multipliers **less than** 1

Multipliers are commutative – an overall multiplier effect can be calculated by combining the multipliers separately

eg Increase of 10% then a reduction of 10%: x 1.10 x 0.9 → x 0.99 The multiplier

Growth and decay

Compound growth | Compound decay



Compound growth and compound decay are exponential graphs

Decay – the values get closer to 0
The constant multiplier is less than one

Growth – the values increase exponentially
The constant multiplier is more than one

Find the original value

Percentage calculations

Original amount × Multiplier = Final Value

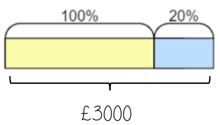
In a test Lucy scored 60% of her questions correctly. Her score was 24. How many questions were on the test.

Original x 0.6 = 24

60% | 24 | Total questions on test

24 ÷ 0.6 = 40 marks
10% = 6
100% = 40

A car sold for a profit £3000 with a profit of 20%. How much was the car originally?



Original x 1.2 = 3000

120% = £3000
10% = £250
100% = £2500

PROPORTION...

Ratio

What do I need to be able to do?

- Equivalent ratios
- Share in a ratio (given total, one part or difference)
- Link ratios and fractions
- Combine a set of ratios
- Share in a ratio (algebraically)
- Solve problems with ratio and algebra
- Ratios and scales

Keywords

Tier 2:
Ratio: a statement of how two numbers compare
Equivalent: of equal value
Proportion: a statement that links two ratios
Fraction: represents how many parts of a whole.
Origin: (0,0) on a graph. The point the two axes cross
Gradient: The steepness of a line
Tier 3:
Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken
Denominator: the number below the line on a fraction. The number represent the total number of parts.
Integer: whole number, can be positive, negative or zero.

Compare with ratio

"For every dog there are 2 cats"

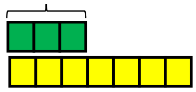
Dogs: Cats
1:2

The ratio has to be written in the same order as the information is given.
e.g. 2:1 would represent 2 dogs for every 1 cat.

Units have the be of the same value to compare ratios

Ratios and fraction

Trees: Flowers
3 : 7



Ratio

Fraction of trees

$\frac{\text{Number of parts of in group}}{\text{Total number of parts}} = \frac{3}{10}$

Fraction

Ratio and scale

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

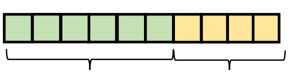
Image : Real life
1cm : 30cm
10cm : 300cm
x 10 x 10

Simplifying a ratio

Cancel down the ratio to its lowest form

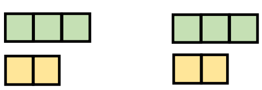
"For every 6 days of rain there are 4 days of sun"

6:4



÷ by 2

3:2



Find the biggest common factor that goes into all parts of the ratio

For 6 and 4 the biggest factor (number that multiples into them is 2)

"For every 3 days of rain there are 2 days of sun" – when this happens twice the ratio becomes 6:4

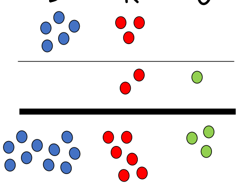
Combining ratios

The ratio of Blue counters to Red counters is 5:3

The ratio of Red counters to Green counters is 2:1

Ratio of Blue to Red to Green

Use equivalent ratios to allow comparison of the group that is common to both statements



10 : 6 : 3

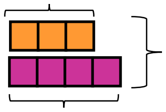
Lowest common multiple of the ratio both statements share

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4
Work out how much each person earns

Model the Question

James: Lucy
3 : 4



Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

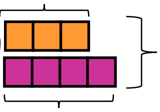
£350 ÷ 7 = £50

$\square$ = one part
= £50

Put back into the question

James: Lucy
x 50 3 : 4 x 50
£150:£200

James = 3 x £50 = £150



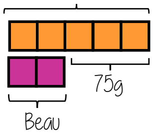
Lucy = 4 x £50 = £200

Sharing in a ratio given a difference

India and Beau share some popcorn in the ratio of 5:2
If India has 75g more popcorn than Beau, what was the original quantity?

Model the Question

India: Beau
5 : 2



Find the value of one part

Difference: 75g

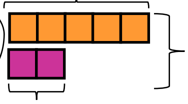
75g ÷ 3 = 25g

$\square$ = one part
= 25g

Put back into the question

India: Beau
x 25 5 : 2 x 25
125g : 50g

India = 5 x 25g = 125g



Beau = 2 x 25g = 50g

Ratios in 1:n and n:1

Show the ratio 4:20 in the ratio of 1:n

This is asking you to cancel down until the part indicated represents 1

The question states that **this part has to be 1 unit**. Therefore **Divide by 4**

4 : 20
1 : 5

This side has to be divided by 4 too – to keep in **proportion**

the n part does not have to be an integer for this type of question

NUMBER... Work with fractions

What do I need to be able to do?

- Fraction of an amount
- Increase or decrease an amount by a fraction
- Use a fraction to find the whole
- Equivalent fractions and mixed numbers
- Add and subtract fractions
- Multiply and divide fractions
- Solve problems with fractions
- Add and subtract algebraic fractions
- Multiply and divide algebraic fractions
- Simplify algebraic fractions
- Add and subtract more complex algebraic fractions (E)
- Multiply and divide more complex algebraic fractions (E)
- Solve equations with algebraic fractions (E)

Keywords

Tier 2:

Rational: a number that can be made by dividing two integers

Irrational: a number that cannot be made by dividing two integers

Inverse operation: the operation that reverses the action

Quotient: the result of a division

Product: the result of a multiplication

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number

Equivalent: of equal value

Tier 3:

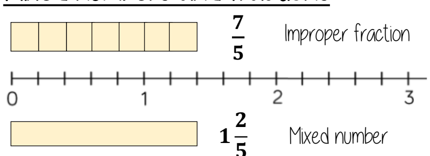
Numerator : the number above the line on a fraction. The top number. Represents how many parts are taken

Denominator: the number below the line on a fraction. The number represent the total number of parts

Mixed numbers: a number with an integer and a proper fraction

Improper fractions: a fraction with a bigger numerator than denominator

Mixed numbers and fractions



$\frac{7}{5}$ Improper fraction

$1\frac{2}{5}$ Mixed number

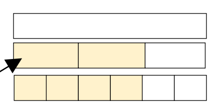
In this model 5 parts make up a whole

Fractions can be bigger than a whole

Equivalent fractions

Numerator and denominator have the same multiplier

$$\frac{2}{3} = \frac{4}{6}$$



$\frac{1}{3} = \frac{2}{6}$

Integers, real and rational numbers

Rational – root word: ratio

Real numbers: $\frac{2}{3}$ stems from 2:1 ($\frac{2}{3}$ of the whole)

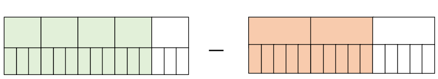
Irrational numbers: $\sqrt{2}$ the solution is a decimal that never ends and does not repeat

The square root of a negative is not a real number and cannot be found

Addition/ Subtraction of fractions

$$\frac{4}{5} - \frac{2}{3}$$

Use equivalent fractions to find a common multiple for both denominators



$$\frac{4}{5} - \frac{2}{3}$$

x 3 x 5

$$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

Multiplication/ Division of fractions

Shade in 3 parts

Repeat it on this many rows

$$\frac{3}{4} \times \frac{2}{3}$$

This many columns

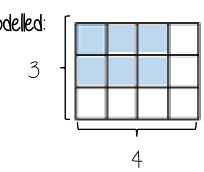
This many rows

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Parts shaded

Total number of parts in the diagram

Modelled:



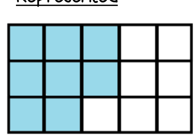
Remember to use reciprocals

$$\frac{2}{5} \div \frac{3}{4}$$

$$\frac{2}{5} \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

Represented

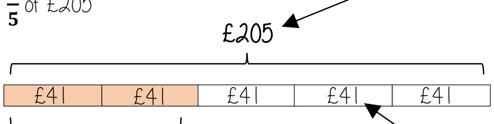


$$= \frac{8}{15}$$

Fraction of a given amount

Find $\frac{2}{5}$ of £205

The bar represents the whole amount

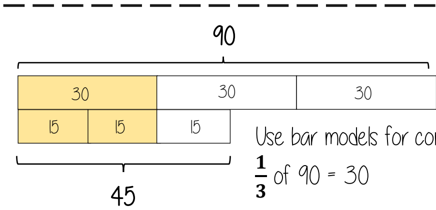


2 out of the 5 equal parts

2 x £41 = <u>£82</u>

£205 ÷ 5 = £41

Each part of the bar model represents £41



Use bar models for comparisons

$\frac{1}{3}$ of 90 = 30

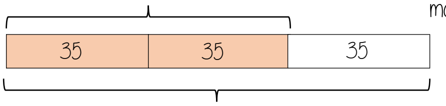
$\frac{2}{3}$ of 45 = 30

$\therefore \frac{1}{3}$ of $90 = \frac{2}{3}$ of 45

Use a fraction of amount

$\frac{2}{3}$ of a value is 70. What is the whole number?

70



$70 \div 2 = 35$

Each part of the bar model represents 35.

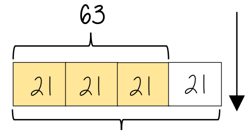
$35 \times 3 = 105$

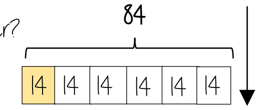
The whole number is 105

The wording of the question is important to setting up the bar model

$\frac{3}{4}$ of a number is 63.

What is $\frac{1}{6}$ of the number?





= 14

Use the whole to find a given part

The reciprocal

When you multiply a number by its reciprocal the answer is always 1

$3 \times \frac{1}{3} = 1$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

The reciprocal of 3 is $\frac{1}{3}$ and vice versa

Reciprocals for division

eg

$5 \div \frac{1}{4} = 20$

$5 \times 4 = 20$

Multiplying by a reciprocal gives the same outcome

Multiplying algebraic fractions(H)

Multiply the **numerators together** and the **denominators together**.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{x}{3} \times \frac{x+2}{x-2}$$

$$= \frac{x(x+2)}{3(x-2)}$$

$$= \frac{x^2+2x}{3x-6}$$

Dividing algebraic fractions(H)

Multiply the first fraction by the **reciprocal of the second fraction**.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

$$\frac{x}{3} \div \frac{2x}{7}$$

$$= \frac{x}{3} \times \frac{7}{2x}$$

$$= \frac{7x}{6x} = \frac{7}{6}$$

Common misconceptions

•*Not recognising double negatives when subtracting algebraic fractions*

Some students fail to recognise the double negative when subtracting fractions in a question such as this.

$$\frac{2x}{7} - \frac{x-1}{7}$$

For example, a student may incorrectly write the following working out.

$$\frac{2x}{7} - \frac{x-1}{7} = \frac{2x-x-1}{7} = \frac{x-1}{7}$$ ✗

In these steps the student has failed to subtract both of the terms of the numerator of the second fraction. To avoid this mistake, it is advisable to write the numerators in brackets, as this helps to remind you that all the terms in the bracket need to be subtracted.

The correct solution is: $\frac{2x}{7} - \frac{x-1}{7} = \frac{(2x)-(x-1)}{7} = \frac{2x-x+1}{7} = \frac{x+1}{7}$

Simplifying algebraic fractions(H)

Factorise the numerator and denominator and **cancel common factors**.

$$\frac{x^2+x-6}{2x-4}$$

$$= \frac{(x+3)(x-2)}{2(x-2)}$$

$$= \frac{x+3}{2}$$

Adding/subtracting algebraic fractions(H)

For $\frac{a}{b} \pm \frac{c}{d}$, **the common denominator** is bd

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad}{bd} \pm \frac{bc}{bd} = \frac{ad \pm bc}{bd}$$

$$\frac{5}{x+2} + \frac{3}{x-2}$$

$$\frac{5(x+2)}{(x+2)(x-2)} + \frac{3(x+2)}{(x+2)(x-2)}$$

$$= \frac{5x-10}{(x+2)(x-2)} + \frac{3x+6}{(x+2)(x-2)}$$

$$= \frac{5x-10+3x+6}{(x+2)(x-2)}$$

$$= \frac{8x-4}{(x+2)(x-2)}$$

$$= \frac{4(2x-1)}{(x+2)(x-2)}$$

NUMBER...

Non-calculator methods

What do I need to be able to do?

- Place value for integers and decimals
- Compare and order numbers
- Add and subtract integers and decimals
- Multiply and divide integers and decimals
- Four operations with directed number
- Order of operations
- Convert recurring decimals to fractions
- Convert more complex recurring decimals to fractions (E)

Keywords

Tier 2:

Interval: between two points or values

Negative: Any number less than zero; written with a minus sign.

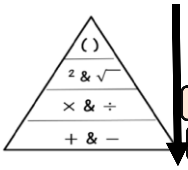
Place holder: We use 0 as a place holder to show that there are none of a particular place in a number

Tier 3:

Integer: a whole number that is positive or negative

Place value: The value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

Order of operations



Brackets

Indices or roots

Multiplication or division

Addition or subtraction

If you have multiple operations from the same tier work from left to right

eg 10 – 3 + 5 → 10 - 3 → 7 + 5

6 x 4 + 8 x 2

24 + 16 = 40

Integer Place Value

Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O
		3	1	4	8	0	3	3	0	2	9

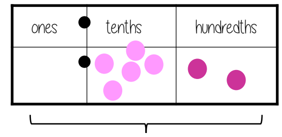
Placeholder

Three billion, one hundred and forty eight million, thirty three thousand and twenty nine

I billion 1, 000, 000, 000

I million 1 000, 000

Decimals



0 ones, 5 tenth and 2 hundredths

0 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.01 + 0.01

= 0 + 0.5 + 0.02

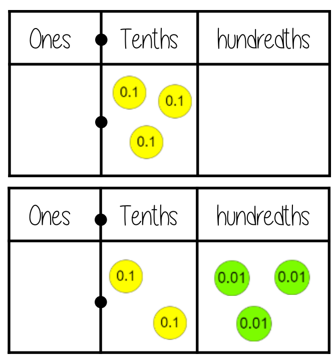
= 0.52

We say "nought point five two"

Five tenths and two hundredths

Comparing decimals

Which the largest of 0.3 and 0.23?



0.3 > 0.23

"There are more counters in the furthest column to the left"

0.30

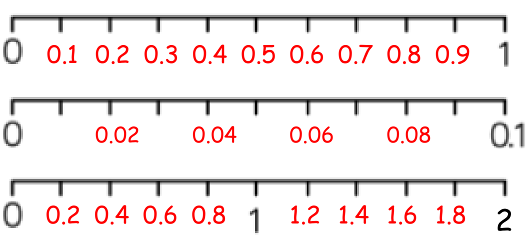
0.23

Comparing the values both with the same number of decimal places is another way to compare the number of tenths and hundredths

Decimal intervals on a number line

One whole spit into 10 parts makes tenths = 0.1

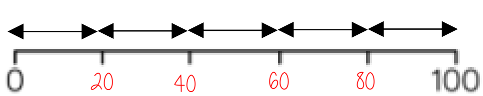
One tenth split into 10 parts makes hundredths = 0.01



Compare integers using <, >, =, ≠

< less than	Two and a half million	=	2 500 000
> greater than	300 000 000	=	Three billion
= equal to	Six thousand and eighty	<	68 000
≠ not equal to			

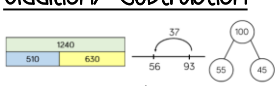
Intervals on a number line



Divide the difference by the number of intervals (gaps).

Eg 100 ÷ 5 = 20

Addition/ Subtraction



Modelling methods for addition/ subtraction

- Bar models
- Number lines
- Part/ Whole diagrams

All give the same solution as represent the same proportion

Multiply the values in proportion until the divisor becomes an integer

Addition is commutative

6 + 3 = 3 + 6

The order of addition does not change the result

Subtraction the order has to stay the same

360 – 147 = 360 – 100 – 40 – 7

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

Formal written methods

	H	T	O
	1	8	7
+	5	4	2

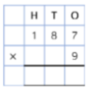
	H	T	O
	4	2	7
−	2	4	9

Remember the place value of each column.

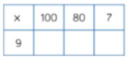
You may need to move 10 ones to the ones column to be able to subtract

Decimals have the same methods remember to align the place value

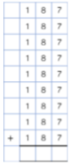
Multiplication methods



Multiplication (column)



Grid method



Repeated addition

Less effective method especially for bigger multiplication

Multiplication with decimals

Perform multiplications as integers

e.g. 0.2 x 0.3 ⟶ 2 x 3

Make **adjustments** to your answer to match the question: 0.2 x 10 = 2

0.3 x 10 = 3

Therefore 6 ÷ 100 = 0.06

Division methods

Short division

3584 ÷ 7 = 512

$$\begin{array}{r} 5\ \ 1\ \ 2 \\ 7\,\overline{)\,3\ {}^{3}5\ \ 8\ {}^{1}4} \end{array}$$

Complex division

÷ 24 = ÷ 6 ÷ 4

Break up the divisor using factors

Division with decimals

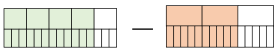
The placeholder in division methods is essential – the decimal lines up on the dividend and the quotient

24 ÷ 0.02 ⟶ 24 ÷ 0.2 ⟶ 240 ÷ 2

Four operations with fractions

Addition and Subtraction

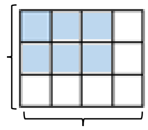
$$\frac{4}{5} - \frac{2}{3}$$



$$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

Multiplication

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$



Division

$$\frac{2}{5} \div \frac{3}{4}$$

$$\frac{2}{5} \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

$$= \frac{8}{15}$$

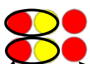
Directed number

● = −1

● = 1

Addition

2 + - 4 = -2



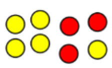
Generalisation

+ − = −

Zero pair (-1 + 1 = 0)

Two " – 1 " left = -2

Subtraction



= 2

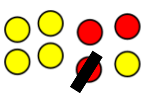
Representation for calculation

Generalisation

- - = +

"Subtract" – means take away or <u>remove</u>

2 - - 1 = 3

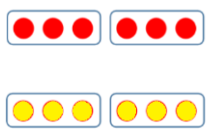


= 3

Take away one ●

Start with the representation of 2

Multiplication



The act of making counters into their negative is turning them over

-2 x -3 = 6

Divisions are the inverse operations

a = 5

b = -4

Brackets around negative substitutions helps remove calculation errors

2a – b = 2 x 5 – (-4) = 10 + 4 = 14

Order of operations

Brackets Operations in brackets are calculated first

Other operations e.g. powers, roots,

Multiplication/ Division

They are carried out in the order from left to right in the question

Addition/ Subtraction

They are carried out in the order from left to right in the question

Calculations with money

Debit - You have £0 or more in an account

Credit - You have less than £0 in an account

Money calculations are to 2dp

Using a calculator – ensure you are working in the correct units.

£1.30 + 50p = 130 + 50 (in pence)

= 1.30 + 0.50 (in pounds)

£1 = 100p

Recurring decimal to fraction(H)

To convert a recurring decimal to a fraction, first **identify the number of places that are recurring**

Ex1 Convert $0.\dot{5}$ to a fraction

Solution

$$\text{Let } x = 0.\dot{5}$$

$$10x = 5.\dot{5}$$

$$9x = 5$$

÷9

$$x = \frac{5}{9}$$

$$\begin{array}{r} 5.\cancel{5} \\ -\ 0.\cancel{5} \\ \hline 5.0 \end{array}$$

Ex2 Convert $0.\dot{1}\dot{2}$ to a fraction

Solution

$$\text{Let } x = 0.\dot{1}\dot{2}$$

$$100x = 12.\dot{1}\dot{2}$$

÷99

$$x = \frac{12}{99} = \frac{4}{33}$$

$$\begin{array}{r} 12.\cancel{12} \\ -\ 0.\cancel{12} \\ \hline 12.00 \end{array}$$

Ex3 Convert $0.\dot{4}2\dot{7}$ to a fraction

Solution

$$\text{Let } x = 0.\dot{4}2\dot{7}$$

$$1000x = 427.\dot{4}2\dot{7}$$

$$999x = 427$$

÷999

$$x = \frac{427}{999}$$

$$\begin{array}{r} 427.\cancel{427} \\ -\ 0.\cancel{427} \\ \hline 427.000 \end{array}$$

GRAPHS...

Straight line graphs

What do I need to be able to do?

- Plot straight line graphs
- Find solutions to equations using straight line graphs
- y = mx + c
- Find the equation of a line from a graph
- Represent solutions to single inequalities on a graph
- **Represent solutions to multiple inequalities on a graph (E)**
- Find the midpoint of a line segment
- Equation of a straight-line graph given one point and a gradient
- Equation of a straight-line graph given two points (E)
- **Equations of perpendicular lines (E)**
- Real-life straight-line graphs

Keywords

Tier 2:

Gradient: the steepness of a line

Intercept: where two lines cross. The y-intercept: where the line meets the y-axis.

Parallel: two lines that never meet with the same gradient.

Coordinate: a set of values that show an exact position on a graph.

Linear: linear graphs (straight line) – linear common difference by addition/ subtraction

Reciprocal: a pair of numbers that multiply together to give 1.

Perpendicular: two lines that meet at a right angle.

Tier 3:

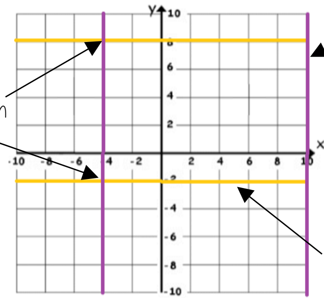
Simultaneous equations: two or more algebraic equations that share common variables and are solved at the same time

Lines parallel to the axes

All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Intersection points



Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

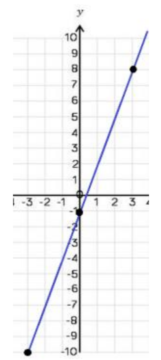
Plotting y = mx + c graphs

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	−3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

y = mx + c

The **coefficient** of x (the number in front of x) tells us the gradient of the line

$y = \frac{1}{2}x - 4$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**

The equation of a line can be rearranged. Eg:

y = c + mx

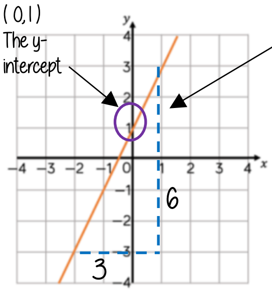
c = y – mx

Identify which coefficient you are identifying or comparing

The **greater** the gradient – the **steeper** the line

Find the equation from a graph

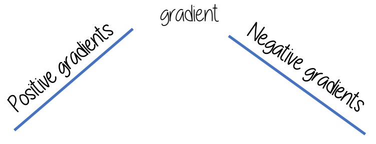
(0,1) The y-intercept



The Gradient $\frac{6}{3} = 2$

$$y = 2x + 1$$

The direction of the line indicates a positive gradient



Finding the Equation of a Line given a point and a gradient

Substitute in the **gradient (m)** and **point (x, y)** into the equation **$y = mx + c$** and **solve for c**.

Find the equation of the line with gradient 4 passing through (2,7)

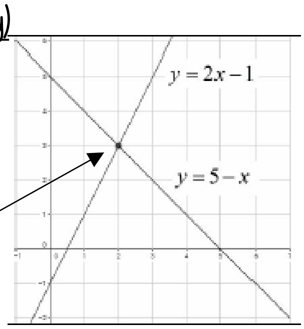
$$y = mx + c$$
$$7 = 4 \times 2 + c$$
$$c = -1$$
$$y = 4x - 1$$

Solving Simultaneous Equations (Graphically)

Draw the graphs of the two equations.

The **solutions will be where the lines meet**. The solution can be written as a **coordinate**.

$y = 5 - x$ and $y = 2x - 1$ meet at the point with coordinates (2,3) so the answer is $x = 2$ and $y = 3$



Finding the Equation of a Line given two points

Use the two points to **calculate the gradient**. Then **repeat the method above** using the gradient and either of the points.

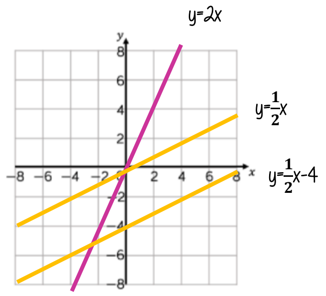
Find the equation of the line passing through (6,11) and (2,3)

$$m = \frac{11 - 3}{6 - 2} = 2$$
$$y = mx + c$$
$$11 = 2 \times 6 + c$$
$$c = -1$$
$$y = 2x - 1$$

Compare Gradients

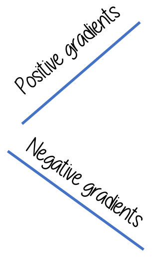
$y = mx + c$

The **coefficient** of x (the number in front of x) tells us the gradient of the line



The **greater** the gradient – the **steeper** the line

Parallel lines have the **same** gradient

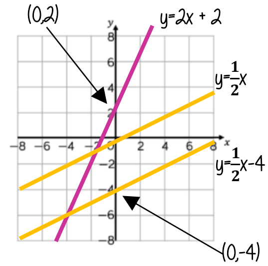


Compare Intercepts

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis

Y intercept



The coordinate of a y intercept will always be (0,c)

Lines with the **same** y-intercept cross in the **same** place

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

The y-intercept shows the minimum charge. The gradient represents the price per mile

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs To represent direct proportion the graph must start at the origin

A box of pens costs £2.30
Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

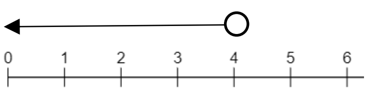
When you have 0 pens this has 0 cost. The gradient shows the price per pen

Represent Inequalities

Multiple methods of representing inequalities

$x < 4$

All values are less than 4

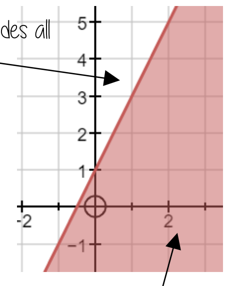


The shaded area indicates all possible values of x

The dotted line shows that the inequality does not include these points

The solid line shows that the inequality includes all the points on this line

$y \geq 2x + 1$



The shaded area indicates all possible solutions to this inequality

PROPORTION...

Probability

What do I need to be able to do?

- Find the probability of a single event
- Use the property that probabilities sum to 1
- List and count outcomes
- Relative frequency
- Sample spaces for 1 or more events
- Two-way tables and frequency trees
- Independent events
- Tree diagrams for independent events
- Tree diagrams for dependent events
- **Conditional probability (Tree diagrams) (E)**

Keywords

Tier 2:

Event: one or more outcomes from an experiment

Outcome: the result of an experiment.

Intersection: elements (parts) that are common to both sets

Union: the combination of elements in two sets.

Expected Value: the value/ outcome that a prediction would suggest you will get

Systematic: ordering values or outcomes with a strategy and sequence

Product: the answer when two or more values are multiplied together.

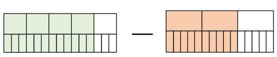
Tier 3:

Universal Set: the set that has all the elements

Add, Subtract and multiply fractions

Addition and Subtraction

$$\frac{4}{5} - \frac{2}{3}$$



$$\underbrace{\frac{12}{15} - \frac{10}{15}}_{} = \frac{2}{15}$$

Use equivalent fractions to find a common multiple for both denominators

Multiplication

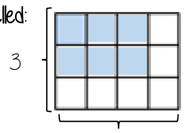
$$\frac{3}{4} \times \frac{2}{3}$$

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Parts shaded

Total number of parts in the diagram

Modelled:



Likeliness of a probability

Impossible	Even chance	Certain
0 or 0%	0.5, $\frac{1}{2}$ or 50%	1 or 100%

The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)

Sum to 1

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$

∴The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

Experimental data

Theoretical probability — What we expect to happen

Experimental probability — What actually happens when we try it out

The more trials that are completed the closer experimental probability and theoretical probability become

The probability becomes more accurate with more trials.

Theoretical probability is proportional

Sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

P (Even number and tales) = $\frac{3}{12}$

Independent events

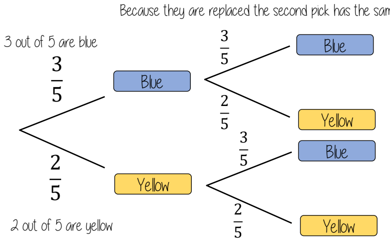
The outcome of two events happening. The outcome of the first event has no bearing on the outcome of the other

$$P\ (A \text{ and } B) = P\ (A) \times P\ (B)$$

Tree diagram for independent event

Isobel has a bag with 3 blue counters and 2 yellow. She picks a counter and replaces it before the second pick.

Because they are replaced the second pick has the same probability



$= P\ (BB)\quad \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

$= P\ (BY)\quad \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

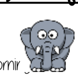
$= P\ (YB)\quad \frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

$= P\ (YY)\quad \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

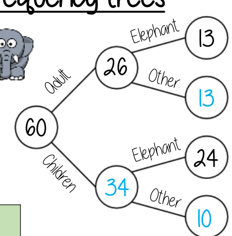
Sum of probabilities = 1

Tables, Venn diagrams, Frequency trees

Frequency trees



60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant.



Frequency trees and two-way tables can show the same information

The total columns on two-way tables show the possible denominators

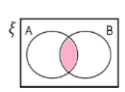
P(adult) = $\frac{26}{60}$

P(Child with favourite animal as elephant) = $\frac{13}{37}$

Two-way table

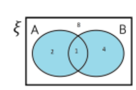
	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Venn diagram



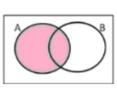
in set A AND set B

$P\ (A \cap B)$



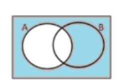
in set A OR set B

$P\ (A \cup B)$



in set A

$P\ (A)$



NOT in set A

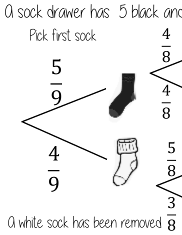
$P\ (A')$

Dependent events(H)

The outcome of the first event has an impact on the second event

Tree diagram for dependent event

A sock drawer has 5 black and 4 white socks, Jamie picks 2 socks from the drawer.



$= P\ (BB)\quad \frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$

$= P\ (BW)\quad \frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$

$= P\ (WB)\quad \frac{4}{9} \times \frac{5}{8} = \frac{20}{72}$

$= P\ (WW)\quad \frac{4}{9} \times \frac{3}{8} = \frac{12}{72}$

Sum of probabilities = 1

NOTE: as "socks" are removed from the drawer the number of items in that drawer is also reduced ∴ the denominator is also reduced for the second pick

NUMBER...

Rounding and estimation

What do I need to be able to do?

:

- Round to decimal places and significant figures
- Estimate answers to calculations
- Use of a calculator
- Error intervals (including truncation)
- Upper and lower bounds

Keywords

Tier 2:

Significant: Place value of importance

Round: Making a number simpler but keeping its value close to what it was.

Decimal: Place holders after the decimal point.

Overestimate: Rounding up – gives a solution higher than the actual value

Underestimate: Rounding down – gives a solution lower than the actual value.

Metric: A system of measurement.

Truncate: to shorten, to shorten a number (no rounding), to shorten a shape (remove a part of the shape)

Round to powers of 10

If the number is halfway between we "**round up**"

5495 to the nearest 1000

5000 6000

5475 to the nearest 100

5400 5500

5475 to the nearest 10

5470 5480

Round to decimal places

2.46192 ← Focus on the numbers **after** the decimal point

"To 1dp" – to one number after the decimal

"To 2dp" – to two numbers after the decimal

2.46192 (to 1dp) - Is this closer to 2.4 or 2.5

2.4 2.5

2.4 | 6192 This shows the number is closer to 2.5

2.46192 (to 2dp) - Is this closer to 2.46 or 2.47

2.46 2.47

2.46 | 192 This shows the number is closer to 2.46

Round to significant figures

SF: Round to the first nonzero number

370 to 1 significant figure is 400

37 to 1 significant figure is 40

3.7 to 1 significant figure is 4

0.37 to 1 significant figure is 0.4

0.00037 to 1 significant figure is 0.0004

372 to 2 significant figures is 370

37.2 to 2 significant figures is 37

3.72 to 2 significant figures is 3.7

0.372 to 2 significant figures is 0.37

0.000372 to 2 significant figures is 0.00037

Estimation

Estimations are useful – especially when using fractions and decimals to check if your solution is possible.

Most estimations round to 1 significant figure

Estimations are useful – especially when using fractions and decimals to check if your solution is possible.

210 + 899 < 1200

This is true because even if both numbers were rounded up, they would reach 300 + 900.

The correct estimation would be

200 + 900 = 1100.

Limits of accuracy

A width w has been rounded to 6.4cm correct to 1dp.

< **6.35** the values would round to 6.3

Error interval

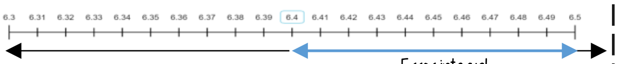
≥ **6.45** the values would round to 6.5

The error interval

$6.35 \leq w < 6.45$

Any value within these limits would round to 6.4 to 1dp

A width w has been truncated to 6.4cm correct to 1dp.



< **6.4** the values would truncate to 6.3

Error interval

≥ **6.5** the values would truncate to 6.5

$6.4 \leq w < 6.5$

Any value within these limits would **truncate** to 6.4 to 1dp

Estimate the calculation

Round to 1 significant figure to estimate

4.2 + 6.7 ≈ 4 + 7 ≈ 11 This is an **overestimate** because the 6.7 was rounded up more

The equal sign changes to show it is an estimation

21.4 x 3.1 ≈ 20 x 3 ≈ 60 This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors.

GEOMETRY...

Perimeter, area and volume

What do I need to be able to do?

- Name 2-D and 3-D shapes
- Perimeter of a 2-D shape
- Area of a 2-D shape
- Area and circumference of a circle
- Arc length and perimeter
- Area of a sector
- Volume of a prism
- Volume of a cylinder
- Nets
- Surface area of a prism
- Surface area of a cylinder

Keywords

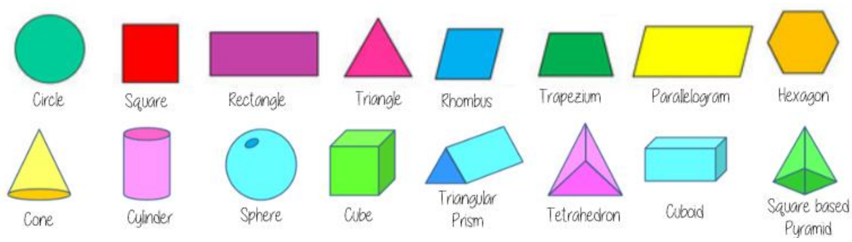
Tier 2:

Edge: a line on the boundary joining two vertex
Face: a flat surface on a solid object
Vertex: a point where two or more line segments meet
Cross-section: a view inside a solid shape made by cutting through it
Plan: a drawing of something when drawn from above (sometimes birds eye view)
Area: the size of the 2D surface
Diameter: the distance from one side of a circle to another through the centre
Radius: the distance from the centre to the circumference of the circle
Tangent: a straight line that touches the circumference of a circle
Chord: a line segment connecting two points on the curve
Surface area: the total area of the surface of a 3D shape.

Tier 3:

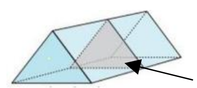
Circumference: the length around the outside of the circle – the perimeter

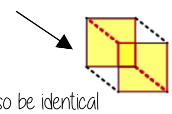
Name 2D & 3D shapes



Recognise prisms

A solid object with two identical ends and flat sides

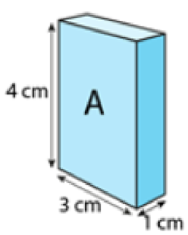


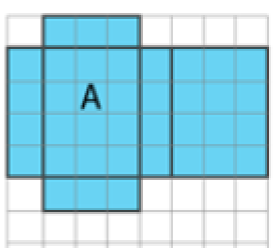


The cross section will also be identical to the end faces.

A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

Nets of cuboids

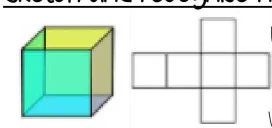




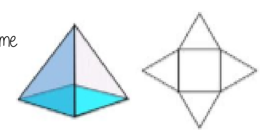
1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

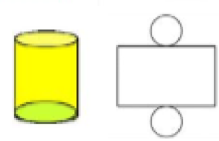
Sketch and recognise nets



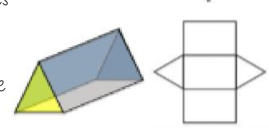
Do they have the same number of faces?



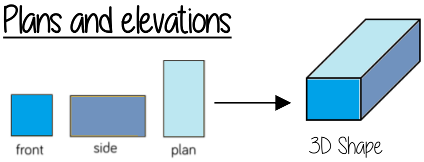
Where do the edges join?



Are the shapes of the faces correct?



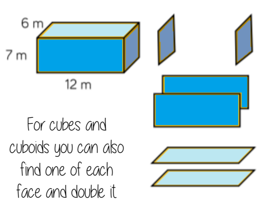
Plans and elevations



The direction you are considering the shape from determines the front and side views

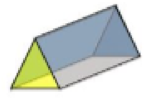
Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area



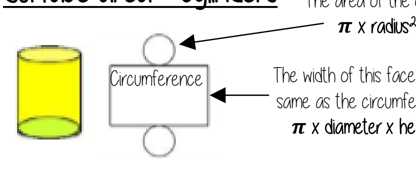
Sides	6 x 7	
	6 x 7	
Front and back	12 x 7	Sum of all sides is surface area
	12 x 7	
Top and Bottom	12 x 6	
	12 x 6	

For cubes and cuboids you can also find one of each face and double it



For other shapes – not all the sides are the same, so calculate the individually

Surface area - cylinders



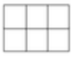
The area of the circle
π x radius^2

The width of this face is the same as the circumference
π x diameter x height

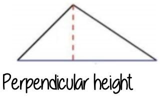
2 x π x radius^2 + π x diameter x height

Area of 2D shapes

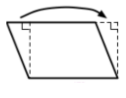
Rectangle
Base x Height



Triangle
½ x Base x Perpendicular height

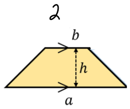


Parallelogram/ Rhombus
Base x Perpendicular height

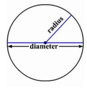


Area of a trapezium

$\frac{(a+b) \times h}{2}$



Area of a circle
π x radius^2



Volumes

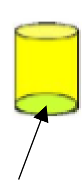
Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space

Counting cubes

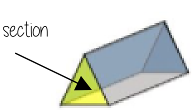
Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape.

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative



Cross section



Cross section

Prisms and cylinders = area cross section x height

Height can also be described as depth

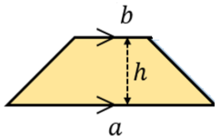
Areas – square units
Volumes – cube units

Areas and volumes can be left in terms of pi π

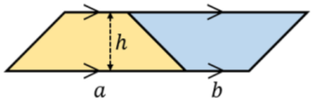
Area of a trapezium

Area of a trapezium

$\frac{(a+b) \times h}{2}$

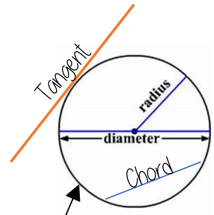


Why?

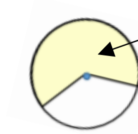


- Two congruent trapeziums make a parallelogram
- New length (a + b) x height
- Divide by 2 to find area of one

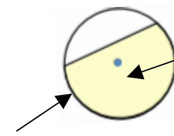
Parts of a circle



Circumference



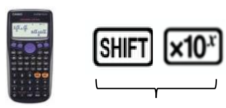
Sector (part of the circle made from two radii)



Segment (part of the circle made from a chord)

An arc is a part of the circumference

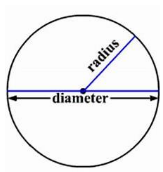
Area of a circle (Calculator)



How to get π symbol on the calculator

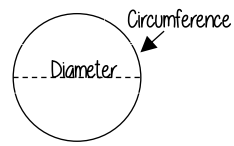
Area of a circle

π x radius^2



It is important to round your answer suitably – to significant figures or decimal places. This will give you a decimal solution that will go on forever!

Circumference



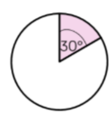
Circumference

π x diameter

Fractional parts of a circle

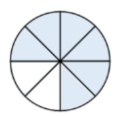
A circle is made up of 360°

Formula to remember:
Area of a circle = πr^2
Circumference of a circle = πd or $2\pi r$



30° represents $\frac{30}{360}$ of a full circle

$\frac{30}{360} = \frac{1}{12}$



$\frac{270}{360}$ of a full circle (in degrees)

$\frac{6}{8}$ of a full circle (in equal parts)

$\frac{3}{4}$ of a full circle

The fraction of the circle is as $\frac{\theta}{360}$

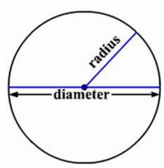
θ represents the degrees in the sector

Area of a circle (Non-Calculator)

Read the question – leave in terms of π or if $\pi \approx 3$ (provides an estimate for answers)

Area of a circle

π x radius^2



Diameter = 8cm

∴ Radius = 4cm

π x radius^2

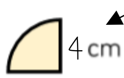
= π x 4^2

= π x 16

= 16π cm^2

Find the area of one quarter of the circle

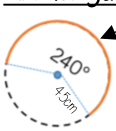
Radius = 4cm



Circle Area = 16π cm^2

Quarter= 4π cm^2

Arc length



Remember an arc is part of the circumference

Circumference of the whole circle = πd = $\pi \times 9 = 9\pi$

Arc length = $\frac{\theta}{360}$ $\times$circumference

$= \frac{240}{360} \times 9\pi$

$= \frac{2}{3} \times 9\pi$ $= 6\pi$

Perimeter

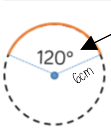
Perimeter is the length around the outside of the shape

This includes the arc length and the radii that encloses the shape

Perimeter = $\frac{\theta}{360}$ $\times$circumference $+ 2r$

$= 6\pi + 9$

Sector area



Remember a sector is part of a circle

Area of the whole circle = πr^2 = $\pi \times 6^2 = 36\pi$

Sector area = $\frac{\theta}{360}$ $\times$ area of circle

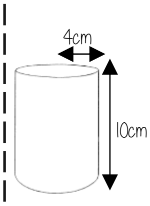
$= \frac{120}{360} \times 36\pi$

$= \frac{1}{3} \times 36\pi$ $= 12\pi$

Volume of a cylinder

Volume Cylinder= $\pi r^2 h$

A cylinder is a prism – cross section is a circle



$V = \pi r^2 h$

$= \pi \times 42 \times 10$

$= \pi \times 160$

$= 160\pi$ cm^2

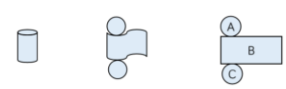
Give your answer in terms of π'

means NOT in terms of pi

$= 502.7$cm^2

Surface area of a cylinder

Surface area cylinder= $2\pi r^2 + \pi dh$



The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles

DATA...

Interpret and represent data

What do I need to be able to do?

- Averages and range
- Averages from an ungrouped frequency table
- Mean from a grouped frequency table
- Averages from a grouped frequency table
- Use data to compare distributions
- Types of data
- Sampling
- Capture and recapture
- Scatter graphs
- Interpolation and extrapolation

Keywords

Tier 2:

Population: the whole group that is being studied

Sample: a selection taken from the population that will let you find out information about the larger group

Random sample: a group completely chosen by change. No predictability to who it will include.

Bias: a built-in error that makes all values wrong by a certain amount

Primary data: data collected from an original source for a purpose.

Secondary data: data taken from an external location. Not collected directly.

Outlier: a value that stands apart from the data set

Tier 3

Interpolation: estimates values within the range of known data

Extrapolation: estimates values outside the range of known data

Averages from lists

The Mean

A measure of average to find the central tendency... a typical value that represents the data

24, 8, 4, 11, 8,

Find the sum of the data (add the values)

55

Divide the overall total by how many pieces of data you have

55 ÷ 5

Mean = 11

The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8,

This can still be easier if it the data is ordered first

Mode = 8

The Median

The value in the center (in the middle) of the data

24, 8, 4, 11, 8,

Put the data in order 4, 8, 8, 11, 24

Find the value in the middle 4, 8, 8, 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

Averages from a table

Non-grouped data

Number of Siblings	0	1	2
Frequency	6	8	6
Subtotal	0	8	12

Overall Frequency: 20

Total number of siblings: 20

The data in a list: 0,0,0,0,0,0,1,1,1,1,1,1,1,1,2,2,2,2,2,2

Mean: $\frac{\text{total number of siblings}}{\text{Total frequency}} = 1$

Grouped data

x Weight(g)	Frequency	Mid Point	MP x Freq
$40 < x \leq 50$	1	45	45
$50 < x \leq 60$	3	65	195
$60 < x \leq 70$	5	65	325

Overall Frequency: 9

Overall Total : 565

Mean: 62.8g

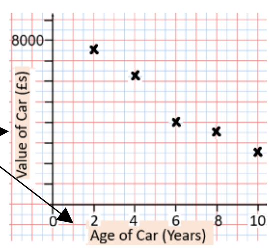
The data in a list: 45, 55, 55, 55, 65, 65, 65, 65, 65

Draw and interpret a scatter graph

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship

All axes should be labelled

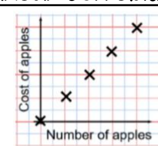


The axis should fit all the values on and be equally spread out

The link between the data can be explained verbally

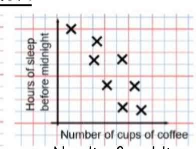
"This scatter graph show as the age of a car increases the value decreases"

Linear Correlation



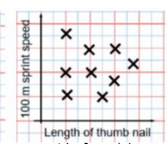
Positive Correlation

As one variable increases so does the other variable



Negative Correlation

As one variable increases the other variable decreases

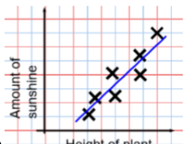


No Correlation

There is no relationship between the two variables

The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph



Things to know:

- The line of best fit DOES NOT need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph

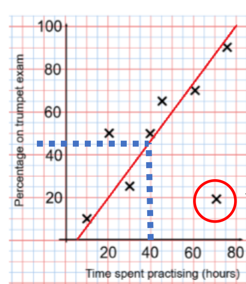
It is only an estimate because the line is designed to be an average representation of the data

It is always a straight line.

Using a line of best fit

Interpolation is using the line of best fit to estimate values inside our data point

e.g. 40 hours revising predicts a percentage of 45



Extrapolation is where we use our line of best fit to predict information outside of our data

This is not always useful – in this example you cannot score more that 100%. So revising for longer can not be estimated

This point is an "outlier"

It is an outlier because it doesn't fit this model and stands apart from the data

Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency.

Mean, mode, median – allows for a comparison about more or less average

Range – allows for a comparison about reliability and consistency of data

GRAPHS...

Non-linear Graphs

What do I need to be able to do?

- Quadratic graphs
- Intercepts and roots of quadratic graphs
- Turning points
- Cubic graphs
- Approximate solutions to equations using graphs
- Equation of the tangent to a curve
- Estimate the area under a curve (E)
- Equation of a circle
- Equation of a tangent to a circle (E)

Keywords

Tier 2:

Quadratic: a curved graph with the highest power being 2. Square power.

Reciprocal: a reciprocal is 1 divided by the number

Exponential: graphs where the variable is a power

Origin: the coordinate (0, 0)

Tier 3:

Cubic: a curved graph with the highest power being 3. Cubic power.

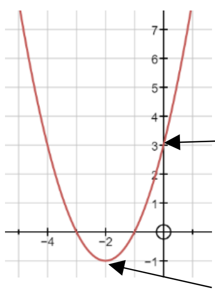
Parabola: a 'u' shaped curve that has mirror symmetry

Quadratic Graphs

$y = x^2 + 4x + 3$

If x^2 is the highest power in your equation then you have a quadratic graph.

It will have a parabola shape



Intersection with the y axis

Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

Coordinate pairs for plotting $(-3, 0)$

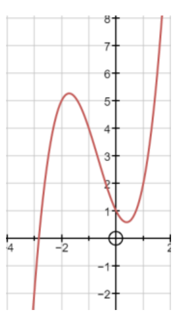
Plot all of the coordinate pairs and join the points with a curve (freehand)

Quadratic graphs are always symmetrical with the turning point in the middle

Interpret other graphs

Cubic Graphs

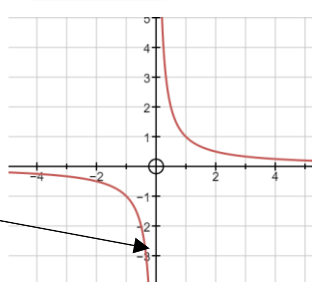
$y = x^3 + 2x^2 - 2x + 1$



If x^3 is the highest power in your equation then you have a cubic graph

Reciprocal Graphs

$y = \frac{1}{x}$

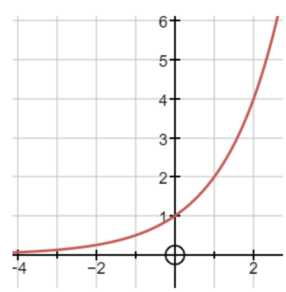


Reciprocal graphs never touch the y axis.
This is because x cannot be 0
This is an asymptote

Exponential Graphs

$y = 2^x$

Exponential graphs have a power of x



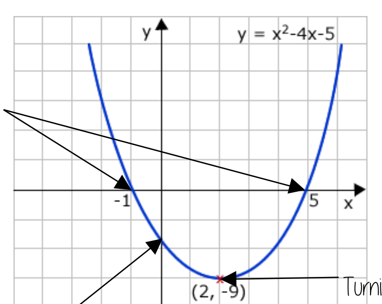
Identify and interpret roots, intercepts of quadratic functions graphically

A root is a **solution**.
The roots of a quadratic are the **x-intercepts of the quadratic graph**

A **turning point** is the point where a quadratic turns.
On a **positive parabola**, the turning point is called a **minimum**.

On a **negative parabola**, the turning point is called a **maximum**.

x- intercepts



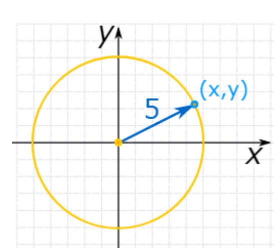
Turning point

y- intercept

Equation of a circle(H)

The equation of a circle, centre (0,0), radius r, is:

$x^2 + y^2 = r^2$

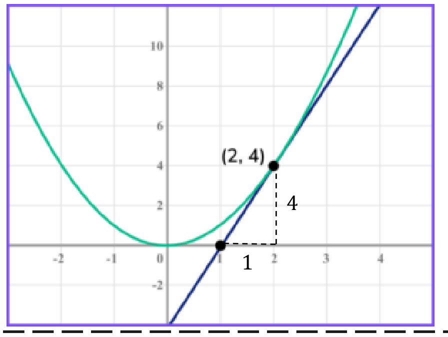


$x^2 + y^2 = 25$

Gradient of a Curve (H)

The gradient of a curve at a point is the same as the gradient of the tangent at that point.

1. Draw a tangent carefully at the point.
2. Make a right-angled triangle.
3. Use the measurements on the axes to calculate the rise and run (change in y and change in x)
4. Calculate the gradient.



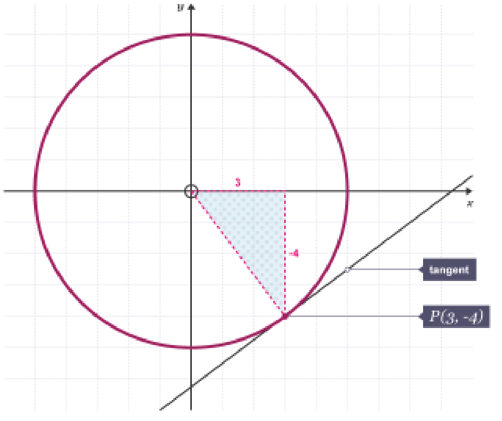
$$Gradient = \frac{Change\ in\ y}{Change\ in\ x}$$

$$= \frac{4}{1} = 4$$

Equation of a tangent to a circle(H)

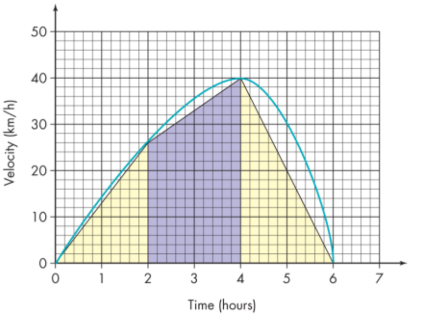
A tangent to a circle at point P is a straight line that touches the circle at P. The tangent is perpendicular to the radius which joins the centre of the circle to the point P.

As the tangent is a straight line, the equation of the tangent will be of the form y = mx + c.
We can use perpendicular gradients to find the value of m, then use the coordinates of P to find the value of c in the equation.



Area Under a Curve (H)

To find the area under a curve, split it up into simpler shapes – such as rectangles, triangles and trapeziums – that approximate the area.



GEOMETRY...

Angles

What do I need to be able to do?

- Angles around a point, on a straight line and vertically opposite
- Angles in triangles and quadrilaterals
- Exterior angles of any polygon
- Interior angles of any polygon
- Solve problems with angles in polygons
- Alternate, corresponding and co-interior angles
- Solve problems with angles in parallel lines
- Solve problems with angles and algebra
- Prove geometric facts (E)

Keywords

Tier 2:

Parallel: Straight lines that never meet

Angle: The figure formed by two straight lines meeting (measured in degrees)

Transversal: A line that cuts across two or more other (normally parallel) lines

Sum: Addition (total of all the interior angles added together)

Regular polygon: All the sides have equal length; all the interior angles have equal size.

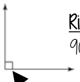
Tier 3:

Polygon: A 2D shape made with straight lines

Isosceles: Two equal size lines and equal size angles (in a triangle or trapezium)

Basic angle rules and notation

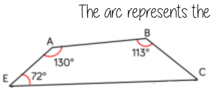
Acute Angles
0°< angle <90°



Right Angles
90°

Right angle notation

The letter in the middle is the angle
The arc represents the part of the angle



Angle Notation: three letters **ABC**
This is the angle at B = 113 °
Line Notation: two letters **EC**
The line that joins E to C.

Obtuse
90°< angle <180°

Reflex
180°< angle <360°

Straight Line
180°

Vertically opposite angles
Equal

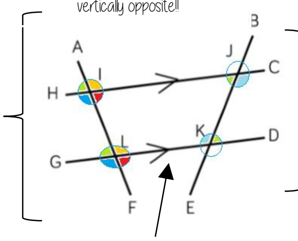
Angles around a point
360 °

Parallel lines

Still remember to look for angles on straight lines, around a point and vertically opposite!!

Lines AF and BE are **transversals** (lines that bisect the parallel lines)

Corresponding angles often identified by their **"F shape"** in position

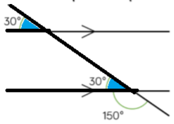


Alternate angles often identified by their **"Z shape"** in position

This notation identifies parallel lines

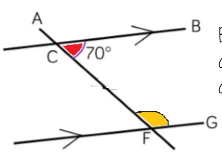
Alternate/ Corresponding angles

Because alternate angles are equal the highlighted angles are the same size



Because corresponding angles are equal the highlighted angles are the same size

Co-interior angles



Because co-interior angles have a sum of 180° the highlighted angle is 110°

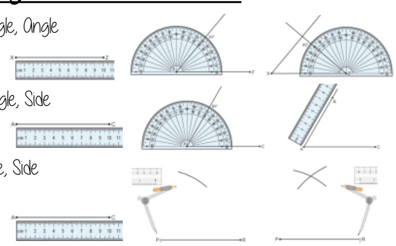
As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Triangles & Quadrilaterals

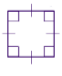
Side, Angle, Angle

Side, Angle, Side

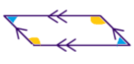
Side, Side, Side



Properties of Quadrilaterals

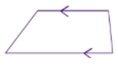


Square
All sides equal size
All angles 90°
Opposite sides are parallel



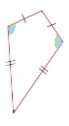
Parallelogram
Opposite sides are parallel
Opposite angles are equal
Co-interior angles

Rectangle
All angles 90°
Opposite sides are parallel



Trapezium
One pair of parallel lines

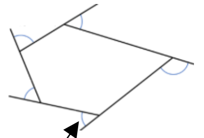
Rhombus
All sides equal size
Opposite angles are equal



Kite
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

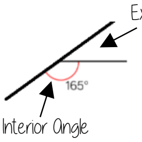
Sum of exterior angles

Exterior angles all add up to 360°



Exterior Angles
Are the angle formed from the straight-line extension at the side of the shape

Using exterior angles



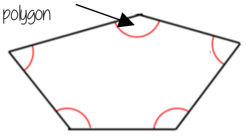
Interior angle + Exterior angle = straight line = 180°
Exterior angle = 180 – 165 = 15°

Number of sides = 360° ÷ exterior angle
Number of sides = 360 ÷ 15 = 24 sides

Sum of interior angles

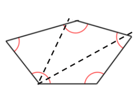
(number of sides – 2) x 180

Interior Angles
The angles enclosed by the polygon



This is an **irregular** polygon – the sides and angles are different sizes

Sum of the interior angles = (5 – 2) x 180

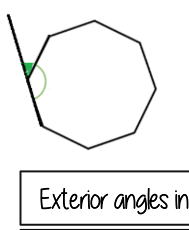


This shape can be made from three triangles
Each triangle has 180°

Sum of the interior angles = 3 x 180
= 540°

Remember this is **all** of the interior angles added together

Missing angles in regular polygons



Exterior angle = 360 ÷ 8 = 45°

Interior angle = $\frac{(8-2) \times 180}{8} = \frac{6 \times 180}{8}$ = 135°

Exterior angles in regular polygons = 360° ÷ number of sides

Interior angles in regular polygons = $\frac{(\text{number of sides} - 2) \times 180}{\text{number of sides}}$

DATA...

Graphs & diagrams

What do I need to be able to do?

- Pictograms
- Line and bar charts
- Dual and composite bar charts
- Draw pie charts
- Interpret pie charts
- Time-series graphs
- Frequency polygons
- Stem-and-leaf diagrams
- **Draw histograms**
- **Interpret histograms**
- **Draw cumulative frequency diagrams**
- **Interpret cumulative frequency diagrams**
- **Box plots**
- **Compare distributions using box plots (E)**

Keywords

Tier 2:

Population: the whole group that is being studied

Sample: a selection taken from the population that will let you find out information about the larger group

Representative: a sample group that accurately represents the population

Random sample: a group completely chosen by change. No predictability to who it will include.

Primary data: data collected from an original source for a purpose.

Secondary data: data taken from an external location. Not collected directly.

Outlier: a value that stands apart from the data set

Quartiles: values that divide a list of numbers into quarters

Tier 3:

Cumulative Frequency: a running total

Histogram: a graphical display of data using bars of different heights

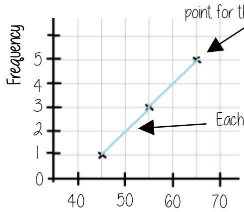
Frequency tables and polygons

$$\text{Volume Sphere} = \frac{4}{3}\pi r^3$$

We do not know from grouped data where each value is placed so have to use an estimate for calculations

MID POINTS

Mid-points are used as estimated values for grouped data. The middle of each group



Each point is plotted at them mid point for the group it represents

Each point is connected with a straight line

The data about weight starts at 40. So the axis can start at 40

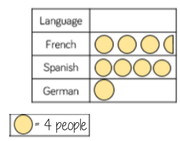
Mid-point

$\frac{\text{Start point} + \text{End point}}{2}$

Pictograms, bar and line charts

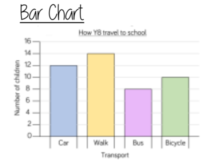
Represents quantitative data

Pictogram



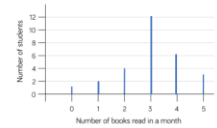
- Need to remember a key
- Visually able to identify mode

Bar Chart



- Gaps between the bars
- Clearly labelled axes
- Scale for the axes
- Title for the bar chart
- Discrete Data

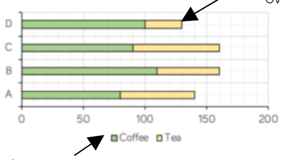
Line Chart



- Gaps between the lines
- Clearly labelled axes
- Scale for the axes
- Discrete Data

Bar and line charts

Composite bar charts



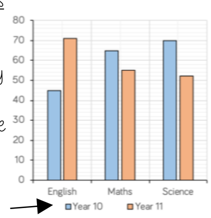
Compare the bars green compared to yellow. The size of each bar is the frequency. Overall total easily comparable

Categories clearly indicated

Dual bar charts

Bars are compared side by side

Easier to compare subgroups



Categories clearly indicated

Draw and interpret Pie Charts

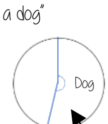
Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$ "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192°$



Use a protractor to draw This is 192°

Multiple method

As 60 goes into 360 – 6 times.

Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

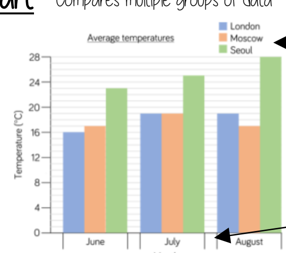
Comparing Pie Charts:

You NEED the overall frequency to make any comparisons

Multiple Bar chart

Compares multiple groups of data

- Clearly labelled axes
- Scale for axes
- Comparable data bars drawn next to each other



Key/ Colour code for separate groups of information

Gap between different categories of data

Stem and leaf

a way to represent data and use to find averages

This stem and leaf diagram shows the age of people in a line at the supermarket.

0	7 9
1	4 5 6 8 8
2	1 3
3	0

Key: 1|4 Means 14 years old

Stem and leaf diagrams:

Must include a key to explain what it represents

The information in the diagram should be ordered

Back to back stem and leaf diagrams

Girls		Boys
5	14	
7, 5, 5, 5, 4	15	3, 8, 9
8, 4, 2, 1, 0	16	2, 5, 7, 7, 7, 8, 8, 9
9, 8, 7, 6, 6, 4, 2, 1, 1, 0, 0	17	0, 2, 3, 6, 6, 7, 7
	18	0, 1, 4, 5

15 | 3,

Means 153 cm tall

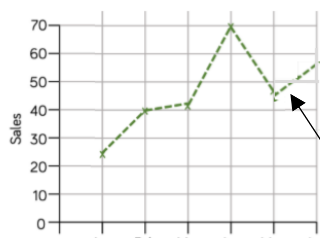
Back to back stem and leaf diagrams

Allow comparisons of similar groups

Allow representations of two sets of data

Time-Series

This time-series graph shows the total number of car sales in £1000 over time



Look for general trends in the data. Some data shows a clear increase or a clear decrease over time.

Readings in-between points are estimates (on the dotted lines). You can use them to make assumptions.

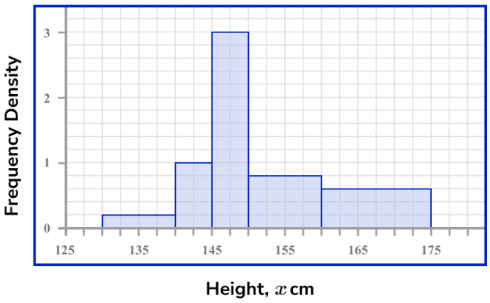
Histograms(H)

A visual way to display frequency data using bars.

Bars can be **unequal in width**.

Height, cm	Frequency	Frequency Density
$130 \le x < 140$	2	0.2
$140 \le x < 145$	5	1
$145 \le x < 150$	15	3
$150 \le x < 160$	8	0.8
$160 \le x < 175$	9	0.6

$$\boldsymbol{Frequency\ Density} = \frac{\boldsymbol{Frequency}}{\boldsymbol{Class\ Width}}$$



Histograms show **frequency density** on the **y-axis**, not frequency.

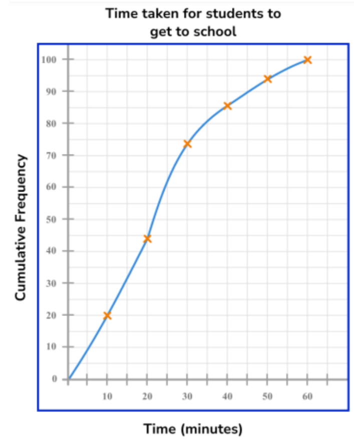
The **area** of the bar is proportional to the **frequency** of that class interval.

$$\boldsymbol{Frequency} = \boldsymbol{Frequency\ Density} \times \boldsymbol{Class\ Width}$$

Cumulative Frequency(H)

Cumulative Frequency is a running total

Time (t minutes)	Frequency	Cumulative Frequency
$0 < t \le 10$	20	20
$10 < t \le 20$	24	$20 + 24 = 44$
$20 < t \le 30$	30	$44 + 30 = 74$
$30 < t \le 40$	12	$74 + 12 = 86$
$40 < t \le 50$	8	$86 + 8 = 94$
$50 < t \le 60$	6	$94 + 6 = 100$



Plot the cumulative frequencies at the end-point of each interval

A cumulative frequency diagram is a curve that goes up. It looks a little like a stretched-out S shape.

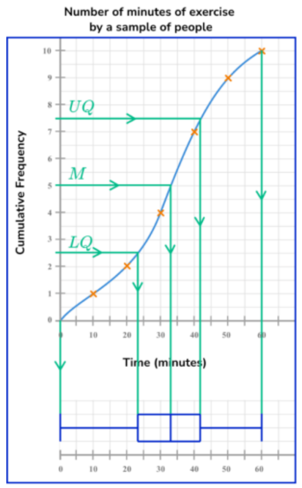
Quartiles and Boxplots(H)

Lower Quartile(Q1): **25%** of the data is less than the lower quartile.

Median(Q2): **50%** of the data is less than the median.

Upper Quartile(Q3): **75%** of the data is less than the upper quartile.

Interquartile Range (IQR): represents the **middle 50%** of the data



Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency.

GEOMETRY...

Vectors

What do I need to be able to do?

- Understand and represent vectors
- Vector notation
- Translate by a vector
- Vectors multiplied by a scalar
- Add vectors
- Add and subtract vectors
- Vector journeys in shapes
- Vectors in quadrilaterals
- Parallel vectors

Keywords

Tier 2:

Direction: the line our course something is going

Magnitude: the magnitude of a vector is its length

Resultant: the vector that is the sum of two or more other vectors

Parallel: straight lines that never meet

Tier 3:

Column vector: a matrix of one column describing the movement from a point

Scalar: a single number used to represent the multiplier when working with vectors

Collinear: lying in the same straight line

Understand and represent vectors

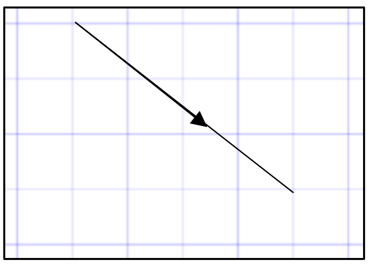
Vectors show both direction and magnitude

Column vectors have been seen in translations to describe the movement of one image onto another

Movement along the x-axis

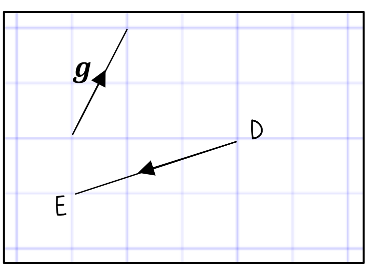
$\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

Movement along the y-axis



The arrow is pointing in the direction from starting point to end point of the vector.	The direction is important to correctly write the vector
The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)	The magnitude stays the same even if the direction changes

Understand and represent vectors



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E.

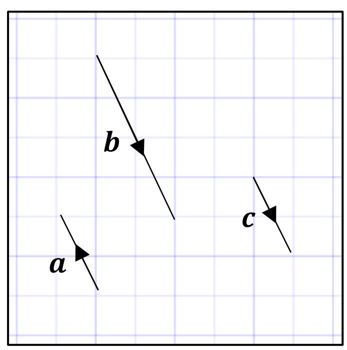
$\overrightarrow{DE} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so $\boldsymbol{g}$ represents the vector $\boldsymbol{g} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$\boldsymbol{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ $\boldsymbol{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\boldsymbol{b} = 2 \times \boldsymbol{c} = 2\boldsymbol{c}$

Multiply $\boldsymbol{c}$ by 2 this becomes $\boldsymbol{b}$. The two lines are parallel

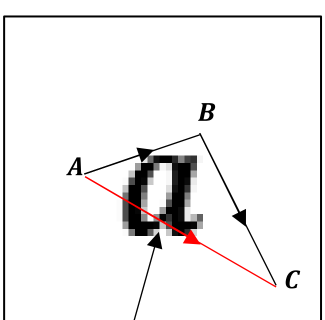
$\boldsymbol{a} = -1 \times \boldsymbol{c} = -\boldsymbol{c}$

The vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are also parallel. A negative scalar causes the vector to reverse direction.

$\boldsymbol{b} = -2 \times \boldsymbol{a} = -2\boldsymbol{a}$

Addition of vectors

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$



The resultant

$$\overrightarrow{AB} + \overrightarrow{BC}$$

$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

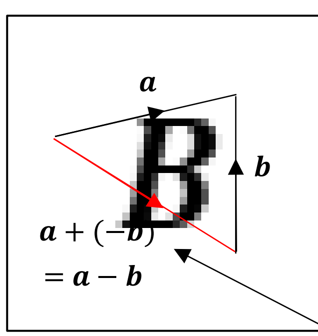
$$= \begin{pmatrix} 3+2 \\ 1+-4 \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Look how this addition compares to the vector $\overrightarrow{AC}$

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Addition and subtraction of vectors



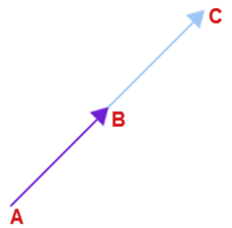
$\boldsymbol{a} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

$$\boldsymbol{a} + (-\boldsymbol{b}) = \begin{pmatrix} 5+ & -0 \\ 1+ & -4 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$

The resultant is $\boldsymbol{a} - \boldsymbol{b}$ because the vector is in the opposite direction to b which needs a scalar of -1

Collinear Vectors(H)

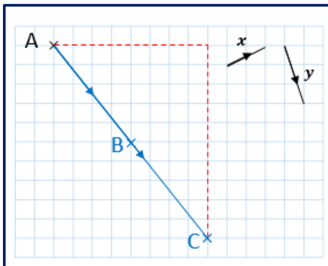
Collinear vectors are vectors that are on the same line.



To show that two vectors are collinear, show that one vector is a multiple of the other (parallel)

AND

that both vectors share a point.



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel

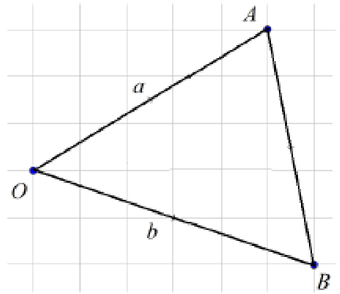
AB and BC both pass through the point B

A, B and C are collinear.

$\overrightarrow{AB} = \boldsymbol{x} + 2\boldsymbol{y} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$\overrightarrow{AC} = 2\boldsymbol{x} + 4\boldsymbol{y} = \begin{pmatrix} 8 \\ -10 \end{pmatrix}$

Vector Geometry(H)

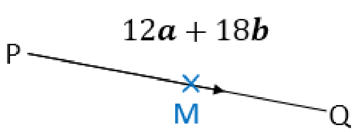


$\overrightarrow{OA} = a \qquad \overrightarrow{AO} = -a$

$\overrightarrow{OB} = b \qquad \overrightarrow{BO} = -b$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b = b - a$

$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -b + a = a - b$

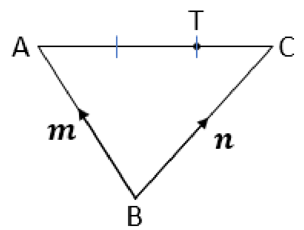


M is the midpoint of PQ.

Find the vector $\overrightarrow{PM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

$$\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ}$$

$$\overrightarrow{PM} = \frac{1}{2}(12\boldsymbol{a} + 18\boldsymbol{b}) \quad = 6\boldsymbol{a} + 9\boldsymbol{b}$$



T lies on AC such that AT : TC = 2 : 1

Find $\overrightarrow{BT}$

$$\overrightarrow{AC} = \boldsymbol{n} - \boldsymbol{m}$$

$$\overrightarrow{BT} = \overrightarrow{BA} + \tfrac{2}{3}\overrightarrow{AC}$$

$$\overrightarrow{BT} = \boldsymbol{m} + \tfrac{2}{3}(\boldsymbol{n} - \boldsymbol{m})$$

$$\overrightarrow{BT} = \boldsymbol{m} + \tfrac{2}{3}\boldsymbol{n} - \tfrac{2}{3}\boldsymbol{m}$$

$$\overrightarrow{BT} = \tfrac{1}{3}\boldsymbol{m} + \tfrac{2}{3}\boldsymbol{n}$$

$$\overrightarrow{BT} = \tfrac{1}{3}(\boldsymbol{m} + 2\boldsymbol{n})$$

NUMBER...

Factors, powers & surds

What do I need to be able to do?

- Factors, multiples and primes
- Square and cube numbers
- Powers and roots
- Negative indices
- Irrational numbers and surds
- Simplify surds (E)
- Expand single brackets with surds (E)
- **Rationalise the denominator**
- **Expand double brackets with surds**
- **Rationalise the denominator with more complex denominators (E)**
- **Solve problems with surds**

Keywords

Tier 2:

Standard (index) Form: A system of writing very big or very small numbers

Base: The number that gets multiplied by a power

Power: The exponent – or the number that tells you how many times to use the number in multiplication

Negative: A value below zero.

Coefficient: The number used to multiply a variable

Tier 3:

Commutative: an operation is commutative if changing the order does not change the result.

Exponent: The power – or the number that tells you how many times to use the number in multiplication

Indices: The power or the exponent.

Surd: The irrational number that is a root of a positive integer, whose value cannot be determined exactly.

Square and cube numbers

Square numbers

1, 4, 9, 16...

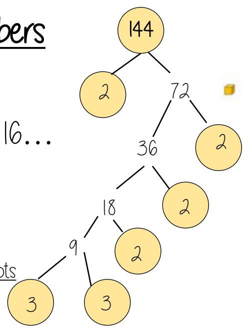
144 = 2 x 2 x 2 x 2 x 3 x 3

2 x 2 x 3 x 2 x 2 x 3

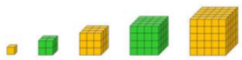
12 x 12

Prime factors can find square roots

$\sqrt{144} = 12$



Cube numbers



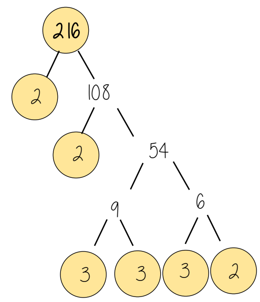
1, 8, 27, 64, 125...

216 = 2 x 2 x 2 x 3 x 3 x 3

2 x 3 x 2 x 3 x 2 x 3

6 x 6 x 6

$\sqrt[3]{216} = 6$



Higher powers and roots

x^n — n – power (number of times multiplied by itself)

x – the base number.

$\sqrt[n]{x}$ — Finding the nth root of any value

Zero and negative indices

$$x^0 = 1$$

Any number divided by itself = 1

$$\frac{a^6}{a^6} = a^6 \div a^6$$

$$= a^{6-6} = a^0 = 1$$

Negative indices do not indicate negative solutions

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

Looking at the sequence can help to understand negative powers

Powers of powers

$$(x^a)^b = x^{ab}$$

$$(2^3)^4 = \underline{2^3 \times 2^3 \times 2^3 \times 2^3}$$

The same base and power is repeated. Use the addition law for indices

$$(2^3)^4 = 2^{12} \qquad a \times b = 3 \times 4 = 12$$

NOTICE the difference

$$(2x^3)^4 = \underline{2x^3 \times 2x^3 \times 2x^3 \times 2x^3}$$

$$(2x^3)^4 = 16x^{12}$$

The addition law applies ONLY to the powers. The integers still need to be multiplied

Fractional indices(H)

The denominator of a fractional power acts as a 'root'.
The numerator of a fractional power acts as a normal power.

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

Other mental strategies for square roots

$$\sqrt{810000} = \sqrt{81} \times \sqrt{10000}$$
$$= 9 \times 100$$
$$= 900$$

Addition/ Subtraction Laws

$a^m \times a^n = a^{m+n}$ $\qquad$ $a^m \div a^n = a^{m-n}$

Rules of Surds(H)

$\sqrt{2}$ is a surd because it is a root which cannot be determined exactly.

$\sqrt{2} = 1.41421356 \ldots$ which never repeats

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \qquad \sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$$

$$a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$$

$$2\sqrt{5} + 7\sqrt{5} = 9\sqrt{5}$$

$$\sqrt{a} \times \sqrt{a} = a \qquad \sqrt{7} \times \sqrt{7} = 7$$

Rationalising the denominator(H)

Rationalising surds is where we convert the denominator of a fraction from an irrational number to a rational number.

$$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{6}}{2}$$

A conjugate contains the same numbers, but with the **opposite sign in the middle**.

In more complex cases, it is useful to multiply the denominator by its conjugate to cancel out the surds in the denominator.

$$\frac{6}{3+\sqrt{7}} = \frac{6(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})}$$

$$= \frac{18-6\sqrt{7}}{9-7} = \frac{18-6\sqrt{7}}{2}$$

$$= 9 - 3\sqrt{7}$$

SIMILARITY...

Pythagoras & trigonometry

What do I need to be able to do?

- Pythagoras' theorem (find the hypotenuse)
- Pythagoras' theorem (find any side)
- Identify hypotenuse, opposite and adjacent sides
- Ratios in right-angled triangles
- Use trigonometric ratios to find an unknown side length
- Use trigonometric ratios to find an unknown angle
- Exact trigonometrical values (E)
- Trigonometry in 3-D shapes
- Area of a non-right-angled triangle
- Use the sine rule to find an unknown length
- Use the sine rule to find an unknown angle
- Use the cosine rule to find an unknown length
- Use the cosine rule to find an unknown angle

Keywords

Tier 2:

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Constant: a value that remains the same

Inverse: function that has the opposite effect.

Tier 3:

Scale Factor: the multiplier of enlargement

Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.

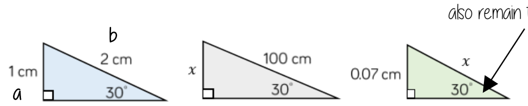
Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.

Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side

Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same



a : b
1 : 2

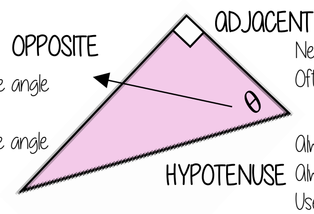
$\frac{a}{b} = \frac{1}{2}$

a : b
x : 100
50 : 100

a : b
0.07 : x
0.07 : 0.14

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE: Always opposite an acute angle. Useful to label second. Position depend upon the angle in use for the question

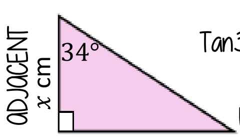
ADJACENT: Next to the angle in question. Often labelled last

HYPOTENUSE: Always the longest side. Always opposite the right angle. Useful to label this first

Tangent ratio: side lengths

$\text{Tan}\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



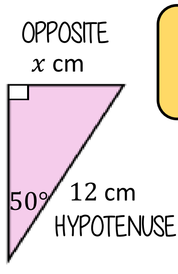
$\text{Tan}34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times \text{Tan}34 = 10$

$x = \frac{10}{\text{Tan}34} = 14.8\text{cm}$

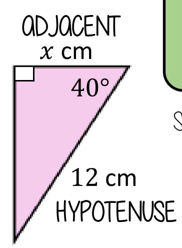
Sin and Cos ratio: side lengths



$\text{Sin}\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE

The Sin(x) ratio is the same as the Cos(90-x) ratio



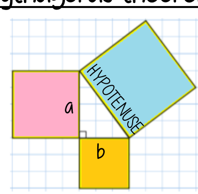
$\text{Cos}\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem

50°



$\text{Hypotenuse}^2 = a^2 + b^2$

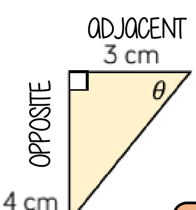
This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras

- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\theta = \text{Tan}^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$

$\theta = \text{Sin}^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = \text{Cos}^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$

$\text{Tan}\theta = \frac{3}{4}$

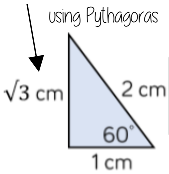
$\theta = \text{Tan}^{-1}\frac{3}{4}$

$\theta = 36.9°$

Key angles

This side could be calculated using Pythagoras

Because trig ratios remain the same for similar shapes you can generalise from the following statements



$\text{Tan}30 = \frac{1}{\sqrt{3}}$

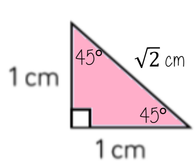
$\text{Tan}60 = \sqrt{3}$

$\text{Cos}30 = \frac{\sqrt{3}}{2}$

$\text{Cos}60 = \frac{1}{2}$

$\text{Sin}30 = \frac{1}{2}$

$\text{Sin}60 = \frac{\sqrt{3}}{2}$



$\text{Tan}45 = 1$

$\text{Cos}45 = \frac{1}{\sqrt{2}}$

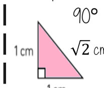
$\text{Sin}45 = \frac{1}{\sqrt{2}}$

Key angles 0° and 90°

$\text{Tan}0 = 0$

~~Tan90~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



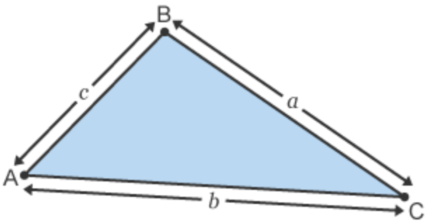
$\text{Sin}0 = 0$

$\text{Sin}90 = 1$

$\text{Cos}0 = 1$

$\text{Cos}90 = 0$

The Sine Rule (H)



The angles are labelled with capital letters. The opposite sides are labelled with lower case letters.
Notice that an angle and its opposite side are the same letter.

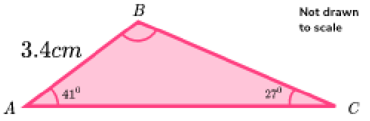
The sine rule is: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

This version is used to calculate lengths.

It can be rearranged to: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

This version is used to calculate angles.

Calculate the length BC. Write your answer to two decimal places.



1. Label each angle (A, B, C) and each side (a, b, c) of the triangle.
2. State the sine rule then substitute the given values into the equation.
3. Solve the equation.

The Cosine Rule (H)

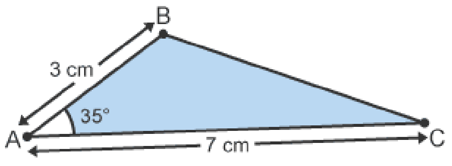
The cosine rule is: $a^2 = b^2 + c^2 - 2bc\ \cos A$

This version is used to calculate lengths.

It can be rearranged to: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

This version is used to calculate angles.

Calculate the length BC. Give the answer to three significant figures.



Use the form $a^2 = b^2 + c^2 - 2bc\cos A$ to calculate the length.

$BC^2 = 3^2 + 7^2 - 2 \times 3 \times 7\cos 35$

$BC^2 = 23.59561414\ldots$

BC = 4.86 cm

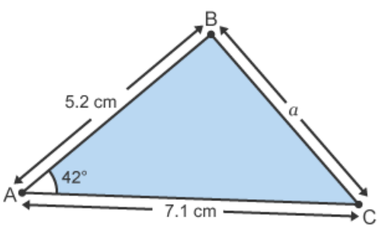
Area of a Triangle (H)

The area of any triangle can be calculated using the formula:

Area of a triangle = $\frac{1}{2}\ ab\ \sin C$

It may be necessary to rearrange the formula if the area of the triangle is given and a length or an angle is to be calculated.

Calculate the area of the triangle. Give the answer to 3 significant figures.



Use the formula:

Area of a triangle = $\frac{1}{2}\ ab\ \sin C$

area= $\frac{1}{2} \times 7.1 \times 5.2 \sin 42$

area = 12.4 cm²

It may be necessary to rearrange the formula if the area of the triangle is given and a length or an angle is to be calculated.

ALGEBRA... Simultaneous Equations

What do I need to be able to do?

- Use one value to find another
- Introduction to simultaneous equations
- Solve simultaneous equations using graphs
- Solve simultaneous equations (no adjustments)
- Manipulating equations
- Solve simultaneous equations (adjust one)
- Solve simultaneous equations (adjust both) (E)
- Solve simultaneous equations by substitution (E)
- **Solve simultaneous equations (one linear, one non-linear) using graphs (E)**
- **Solve simultaneous equations (one linear, one non-linear) by equating expressions (E)**
- **Solve simultaneous equations (one linear, one non-linear) using substitution (E)**

Keywords

Tier 2:

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet.

Equation: an equation says that two things are equal – it will have an equals sign =

Substitute: replace a variable with a numerical value

Eliminate: to remove

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Coordinate: a set of values that show an exact position.

Intersection: the point two lines cross or meet.

Tier 3:

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8

$y = 3x + 5$

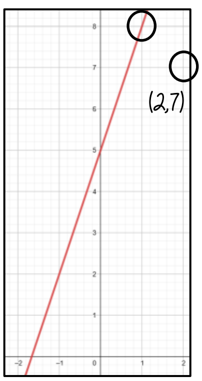
$8 = 3(1) + 5$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5

Is (2,7) on the same line?

$7 \neq 3(2) + 5$

No 7 does NOT equal 6+5



Substituting known variables

A line has the equation 3x + y = 14

Stephanie knows the point x = 4 lies on that line. Find the value for y

Two different variables, two solutions

$x = 4$

$3x + y = 14$ | x | x | x | y | 14

$3(4) + y = 14$ | 4 | 4 | 4 | y | 14

$12 + y = 14$

$-12 \quad -12$

$y = 2$ | y | 2

Substituting in an expression

x = 2y | y y | x

x + y = 30 | x y | 30

Pair of simultaneous equations (two representations)

Substitute 2y in place of the x variable as they represent the same value

$x = 2y$ $\quad$ $x + y = 30$

y y y | 30 $\quad 3y = 30$

$3y = 30$

$\div 3 \quad \div 3$

$y = 10$

$x = 2y$ | 10 10 | x

$x = 20$

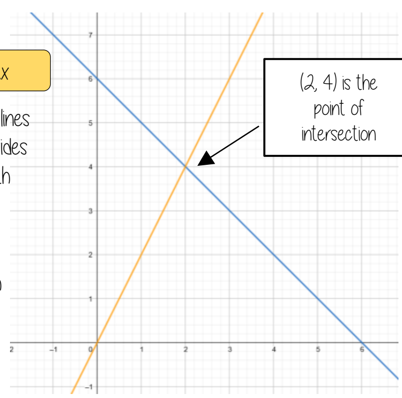
Solve graphically

x + y = 6 $\quad$ y = 2x

Linear equations are straight lines. The point of intersection provides the x and y solution for both equations

The solution that satisfies both equations is

$x = 2$ and $y = 4$



(2, 4) is the point of intersection

Solve by subtraction

18 | x x x y y

10 | x y y

8 | x x

$x = 4$

$y = 3$

$3x + 2y = 18$

$-\ \ x + 2y = 10$

$2x = 8$

$\div 2 \quad \div 2$

$x = 4$

$x + 2y = 10$

$(4) + 2y = 10$

$-4 \quad -4$

$2y = 6$

$\div 2 \quad \div 2$

$y = 3$

x x x y y = 18

x y y = 10

x x = 8

x = 4

y = 3

Solve by addition

$3x + 2y = 16$

$+\ 6x - 2y = 2$

$9x = 18$

$\div 9 \quad \div 9$

$x = 2$

$3x + 2y = 16$

$3(2) + 2(y) = 16$

$6 + 2y = 16$

$-6 \quad -6$

$2y = 10$

$y = 5$

Addition makes zero pairs

x x x y y = 16

x x x -y -y x x x = 2

= 18

x = 2

y = 5

Solve by adjusting one

$h + j = 12$

$2h + 2j = 29$

No equivalent values

12 | h j

29 | h h j j j

$2h + 2j = 24$

$2h + 2j = 29$

24 | h h j j

29 | h h j j j

By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

Solve by adjusting both

$2x + 3y = 39$

$5x - 2y = -7$

= 39

= −7

Use LCM to make equivalent x OR y values. Because of the negative values using zero pairs and y values is chosen choice

$4x + 6y = 78$

$15x - 6y = -21$

Now solve by addition

= 78

= −21

Addition makes zero pairs

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (-3, 9) lie on the curve y = x^2 and the straight line y=2x+15?

This coordinate represents x=-3 and y=9

$y = x^2$

$y = (-3)^2$

$\boldsymbol{y = 9}$

$y = 2x + 15$

$y = 2(-3) + 15$

$y = -6 + 15$

$\boldsymbol{y = 9}$

As the substitution makes the equation correct the coordinate (-3, 9) IS on the curve y = x^2 and the straight line y=2x+15

Solve graphically: linear/quadratic(H)

y = x^2 | y = x + 2

The points of intersection provide the x and y solutions for both equations

Intersection at $(2, 4)$

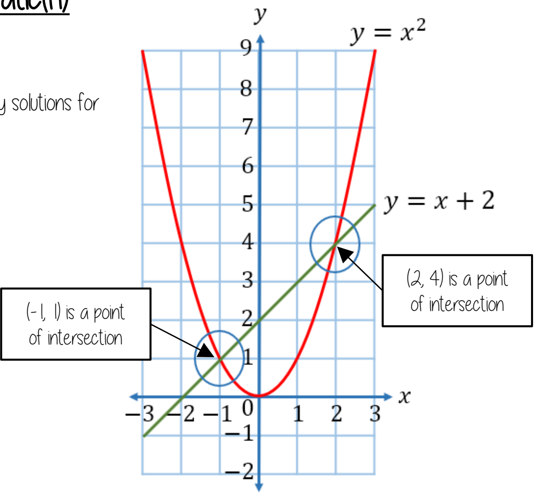
$\boldsymbol{x = 2}$ $\boldsymbol{y = 4}$

Intersection at $(-1, 1)$

$\boldsymbol{x = -1}$ $\boldsymbol{y = 1}$

The solution that satisfies both equations is

$x = 2\,, y = 4$
and
$x = -1\,, y = 1$



Solve by rearranging: linear/quadratic(H)

If $y = x^2$ and $y = x + 12$ then $x^2 = x + 12$

y
x^2
y
$x + 12$

$x^2 = x + 12$

Rearrange to form a quadratic equation equal to zero

$x^2 - x - 12 = 0$

Factorise and solve the resulting linear equations

$(x - 4)(x + 3) = 0$

$\boldsymbol{x = 4\,, x = -3}$

Substitute the values of x into one of the original equations to find the corresponding values for y

$y = x^2$

$y = (4)^2$ $\qquad$ $y = (-3)^2$

$\boldsymbol{y = 16}$ $\qquad$ $\boldsymbol{y = 9}$

The solutions that satisfy both equations are

$\boldsymbol{x = 4}$ **and** $\boldsymbol{y = 16}$

$\boldsymbol{x = -3}$ **and** $\boldsymbol{y = 9}$

Solve by substitution: linear/quadratic(H)

Determine the variable to substitute

$y^2 + x^2 = 16$

$y = x + 4$

Substitute, expand and rearrange to form a quadratic equation equal to zero

$(x + 4)^2 + x^2 = 16$

$x^2 + 8x + 16 + x^2 = 16$

$2x^2 + 8x = 0$

Factorise and solve the resulting linear equations

$2x(x + 4) = 0$

$\boldsymbol{x = 0}$ $\boldsymbol{x = -4}$

Substitute these values of x into one of the original equations to find the corresponding values for y

$y = x + 4$

$y = 0 + 4$ $\qquad$ $y = -4 + 4$

$\boldsymbol{y = 4}$ $\qquad$ $\boldsymbol{y = 0}$

The solutions that satisfy both equations are

$\boldsymbol{x = 0}$ **and** $\boldsymbol{y = 4}$

$\boldsymbol{x = -4}$ **and** $\boldsymbol{y = 0}$