YEAR 9
KNOWLEDGE ORGANISERS

White Rose Maths

@whisto_maths

NUMBER... Properties of number

What do I need to be able to do?

- Factors, multiples and primes
- Write a number as a product of prime factors
- Use prime factors (E)
- Highest common factor (HCF) and lowest common multiple (LCM)
- Venn diagrams
- Use a Venn diagram to calculate the HCF and LCM
- Integers, real numbers and rational numbers
- Introduction to surds (E)

Keywords

Tier 2:

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number.

Prime: an integer with only 2 factors.

Conjecture: a statement that might be true (based on reasoning) but is not proven.

Counterexample: a special type of example that disproves a statement.

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Tier 3:

HCF: highest common factor (biggest factor two or more numbers share)

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15…

$3x, 6x, 9x$ …

This list continues and doesn't end

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

4.5 is not a multiple of 3 because it is 3 x 1.5

Not an integer

Factors

Arrays can help represent factors

5 x 2 or 2 x 5

Factors of 10
1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions

x x x x x x

$6x \times 1$ OR $6 \times x$

Factors of $6x$
6, x, 1, $6x$, $2x$, 3, $3x$, 2

x x / x x / x x $\quad 2x \times 3$

x x x / x x x $\quad 3x \times 2$

Prime numbers

2

- Integer
- Only has 2 factors
- and itself

The first prime number

The only even prime number

Learn or how-to quick recall…

2, 3, 5, 7, 11, 13, 17, 19, 23, 29…

Common multiples and LCM

Common multiples are multiples two or more numbers share

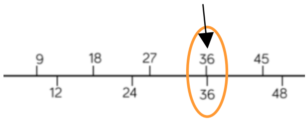
LCM – Lowest common multiple

LCM of 9 and 12

9	9, 18, 27, 36, 45, 54
12	12, 24, 36, 48, 60

LCM = 36

The first time their multiples match



Comparing fractions

$\frac{3}{5}$ and $\frac{7}{10}$

Compare fractions using a LCM denominator

$\frac{6}{10}$ and $\frac{7}{10}$

Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18	1, 2, 3, 6, 9, 18
30	1, 2, 3, 5, 6, 10, 15, 30

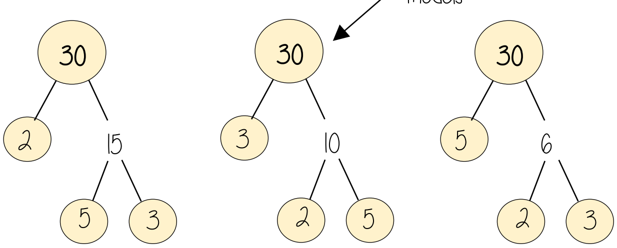
Common factors (factors of both numbers)
1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

Multiplication is commutative

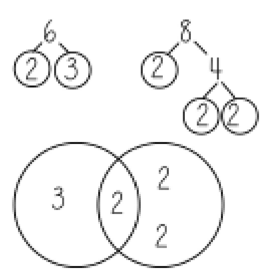
30 = 2 x 3 x 5

Multiplication of prime factors

Using prime factors for predictions

eg 60	30 x 2	2 x 3 x 5 x **2**
150	30 x 5	2 x 3 x 5 x **5**

Venn Diagrams for LCM & HCF



LCM of 6 and 8
= 3 x 2 x 2 x 2
= 24

HCF of 6 and 8 = 2

Integers, real and rational numbers

Rational – root word: ratio

Real numbers: $\frac{2}{3}$ stems from 2:1 ($\frac{2}{3}$ of the whole)

Irrational numbers: $\sqrt{2}$ the solution is a decimal that never ends and does not repeat.

$\sqrt{2}$ is a surd because it is a root which cannot be determined exactly

$\sqrt{2} = 1.41421356$ … which never repeats

The square root of a negative is not a real number and cannot be found.

NUMBER... Percentages

What do I need to be able to do?

- Percentage increase and decrease
- Express a change as a percentage
- Find the original value after a percentage change
- Solve problems with percentages (non-calculator)
- Solve problems with percentages (calculator)
- Repeated percentage change
- Understand interest
- Simple interest
- Compound interest

Keywords

Tier 2:

Percent: parts per 100 – written using the % symbol

Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.

Fraction: a fraction represents how many parts of a whole value you have.

Equivalent: of equal value.

Reduce: to make smaller in value.

Growth: to increase/ to grow.

Invest: use money with the goal of it increasing in value over time (usually in a bank).

Multiplier: the number you are multiplying by.

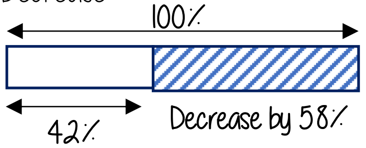
Profit: the income take away any expenses/ costs.

Tier 3:

Integer: whole number, can be positive, negative or zero.

Percentage Increase/ Decrease

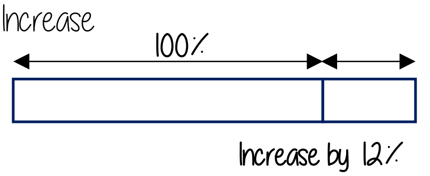
Decrease



Multiplier
Less than 1

$1.00 - 0.58 = 0.42$

Increase

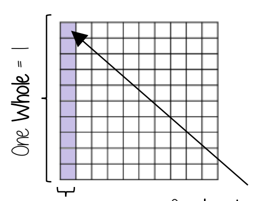


$100\% + 12\% = 112\%$

$1.00 + 0.12 = 1.12$

Multiplier
More than 1

FDP Equivalence

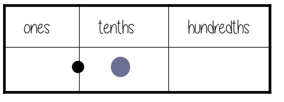


$\frac{10}{100} = \frac{1}{10} = 0.10$

Percentage
100% = a whole
= 100 hundredths

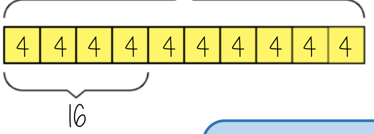
10 hundredths
10 out of 100
10%

One **hundredth**
(one whole split into 100 equal parts)



Reverse Percentages

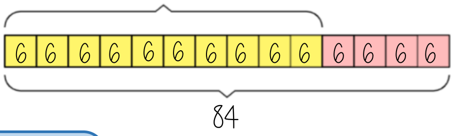
40% of my number is 16. What am I thinking of?



40% = 16
10% - 4
100% = 40

Try to scale down to 10% or 1% and then scale back up to 100%

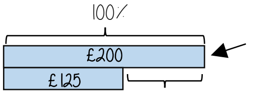
140% of my number is 84. What is the original number?



140% = 84
10% - 6
100% = 60

Percentage change

I bought a phone for £200. A year later sold it for £125.



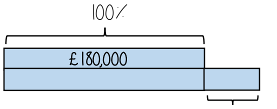
All values of change compare to the ORIGINAL value

Percentage loss

$\frac{75}{200} \times 100 = 37.5\%$

$$\frac{\textit{Difference in values}}{\textit{Original value}} \times 100$$

I bought a house for £180,000, I later sold it for £216,000.



Percentage profit

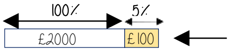
Money made (profit value)

$\frac{36000}{180000} \times 100 = 20\%$

Simple and compound interest

Simple Interest

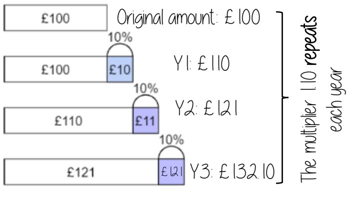
James invests £2000 at 5% simple interest



The original value increases by this amount every year

Compound Interest

Tess invests £100 at 10% compound interest for 3 years



GEOMETRY…

Area and volume

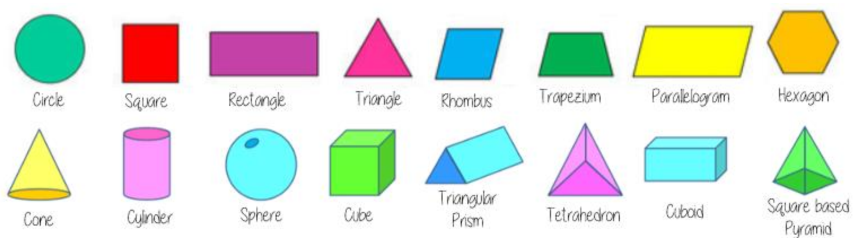
What do I need to be able to do?

- Nets
- Area of a 2-D shape
- Area and circumference of a circle
- Surface area of cubes and cuboids
- Surface area of a triangular prism (E)
- Surface area of a cylinder (E)
- Volume of a prism
- Volume of a cylinder
- Volume of cones, pyramids and spheres (E)
- Convert metric units of area and volume (E)

Keywords

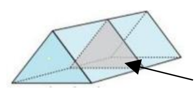
Tier 2:
Edge: a line on the boundary joining two vertex
Face: a flat surface on a solid object
Vertex: a point where two or more line segments meet
Cross-section: a view inside a solid shape made by cutting through it
Plan: a drawing of something when drawn from above (sometimes birds eye view)
Area: the size of the 2D surface
Diameter: the distance from one side of a circle to another through the centre
Radius: the distance from the centre to the circumference of the circle
Tangent: a straight line that touches the circumference of a circle
Chord: a line segment connecting two points on the curve
Surface area: the total area of the surface of a 3D shape.
Tier 3:
Circumference: the length around the outside of the circle – the perimeter

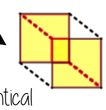
Name 2D & 3D shapes



Recognise prisms

A solid object with two identical ends and flat sides

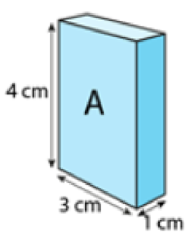


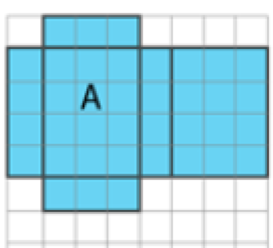


The cross section will also be identical to the end faces.

A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

Nets of cuboids

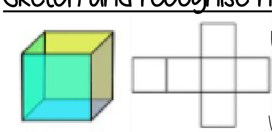




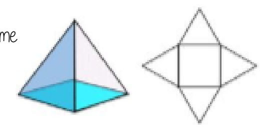
1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

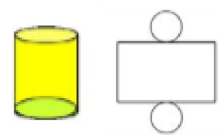
Sketch and recognise nets



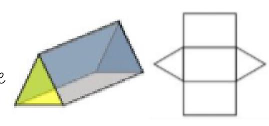
Do they have the same number of faces?



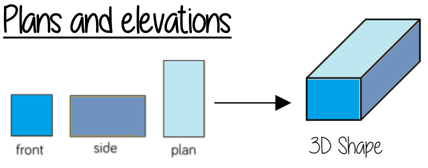
Where do the edges join?



Are the shapes of the faces correct?



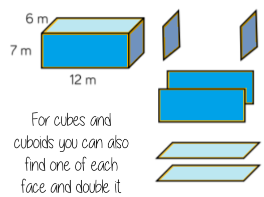
Plans and elevations



The direction you are considering the shape from determines the front and side views

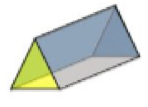
Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area



Sides	6 x 7	Sum of all sides is surface area
	6 x 7	
Front and back	12 x 7	
	12 x 7	
Top and Bottom	12 x 6	
	12 x 6	

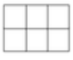
For cubes and cuboids you can also find one of each face and double it



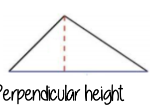
For other shapes – not all the sides are the same, so calculate the individually

Area of 2D shapes

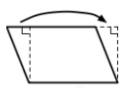
Rectangle
Base x Height



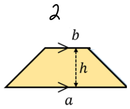
Triangle
½ x Base x Perpendicular height



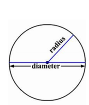
Parallelogram/ Rhombus
Base x Perpendicular height



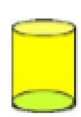
Area of a trapezium
$\frac{(a+b) \times h}{2}$

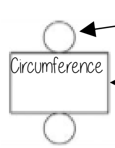


Area of a circle
π x radius²



Surface area - cylinders





The area of the circle
π x radius²

The width of this face is the same as the circumference
π x diameter x height

2 x π x radius² + π x diameter x height

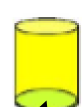
Volumes

Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space

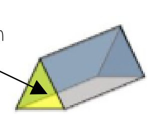
Counting cubes
Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative



Cross section



Cross section

Prisms and cylinders = area cross section x height

Height can also be described as depth

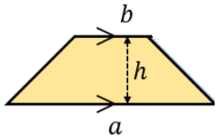
Areas – square units
Volumes – cube units

Areas and volumes can be left in terms of pi π

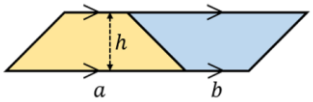
Area of a trapezium

Area of a trapezium

$\frac{(a+b) \times h}{2}$

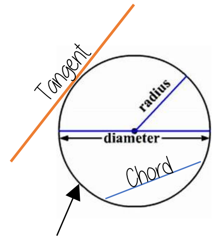


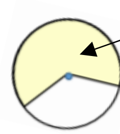
Why?



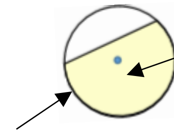
- Two congruent trapeziums make a parallelogram
- New length (a + b) x height
- Divide by 2 to find area of one

Parts of a circle





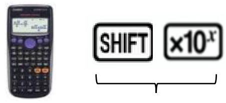
Sector (part of the circle made from two radii)



Segment (part of the circle made from a chord)

An arc is a part of the circumference

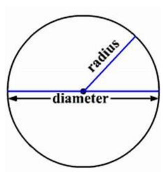
Area of a circle (Calculator)



How to get π symbol on the calculator

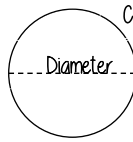
Area of a circle

π x radius^2



It is important to round your answer suitably – to significant figures or decimal places. This will give you a decimal solution that will go on forever!

Circumference



Circumference

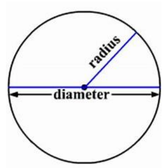
π x diameter

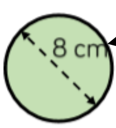
Area of a circle (Non-Calculator)

Read the question – leave in terms of π or if $\pi \approx 3$ (provides an estimate for answers)

Area of a circle

π x radius^2





Diameter = 8cm

∴ Radius = 4cm

π x radius^2

= π x 4^2

= π x 16

= 16π cm^2

Find the area of one quarter of the circle

Radius = 4cm

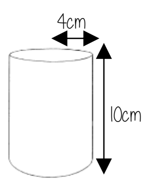
Circle Area = 16π cm^2

Quarter= 4π cm^2

Volume of a cylinder

Volume Cylinder= $\pi r^2 h$

A cylinder is a prism – cross section is a circle



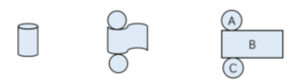
$$V = \pi r^2 h$$
$$= \pi \times 42 \times 10$$
$$= \pi \times 160$$
$$= \underline{160\pi} \text{ cm}^2$$

Give your answer in terms of π' means NOT in terms of pi

$= \underline{502.7}$cm^2

Surface area of a cylinder

Surface area cylinder= $2\pi r^2 + \pi dh$

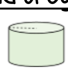


The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles

Volume of a cone and a cylinder

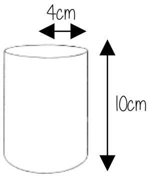
Volume Cylinder= $\pi r^2 h$



A cylinder is a prism – cross section is a circle

Volume Cone = $\frac{1}{3}\pi r^2 h$

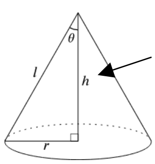
A cone is a pyramid with a circular base



$$V = \pi r^2 h$$
$$= \pi \times 42 \times 10$$
$$= \pi \times 160$$
$$= \underline{160\pi} \text{ cm}^2$$

Give your answer in terms of π' means NOT in terms of pi

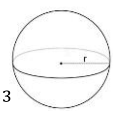
$= \underline{502.7}$cm^2



The height of a cone is the perpendicular height from the vertex to the base

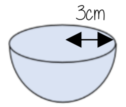
Look out for trigonometry or Pythagoras theorem – the radius forms the base of a right-angled triangle

Volume of a sphere



Volume Sphere = $\frac{4}{3}\pi r^3$

NOTE: This is now a **cubed** value



Volume Sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 3^3$

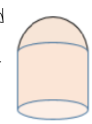
$= \frac{4}{3} \times \pi \times 27$ $= \underline{36\pi}$

A hemisphere is half the volume of the overall sphere

$= 36\pi \div 2$

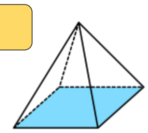
$= 18\pi$

Look out for hemispheres being placed on other 3D shapes, e.g cones and cylinders



Pyramid

Volume Pyramid = $\frac{1}{3}$ *x base area x h*



The surface area of a pyramid can be calculated by adding the area of the base to the sum of the areas of the triangular faces

Convert units of area and volume

Converting units of area and volume is converting between different units from the metric system involving area and volume.

To do this we need to be able to convert between different units of length and then adapt them for area and volume.

Units of length

$mm \xrightarrow{\div 10} cm \xrightarrow{\div 100} m$

$m \xrightarrow{\times 100} cm \xrightarrow{\times 10} mm$

Units of area

$mm^2 \xrightarrow{\div 10^2} cm^2 \xrightarrow{\div 100^2} m^2$

$m^2 \xrightarrow{\times 100^2} cm^2 \xrightarrow{\times 10^2} mm^2$

Units of volume

$mm^3 \xrightarrow{\div 10^3} cm^3 \xrightarrow{\div 100^3} m^3$

$m^3 \xrightarrow{\times 100^3} cm^3 \xrightarrow{\times 10^3} mm^3$

ALGEBRA . Equations, inequalities and formulae

What do I need to be able to do?

- Solve equations and inequalities
- Solve equations and inequalities with brackets
- Inequalities with negative numbers (E)
- Solve equations and inequalities with unknowns on both sides
- Solve problems with equations and inequalities
- Substitute into formulae and equations
- Change the subject of a formula (one-step)
- Change the subject of a formula (two-step)
- Change the subject of complex formula (E)

Keywords

Tier 2:

Inequality: an inequality compares who values showing if one is greater than, less than or equal to another

Variable: a quantity that may change within the context of the problem

Rearrange: Change the order

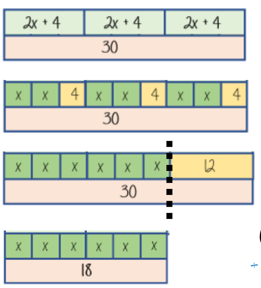
Substitute: replace a variable with a numerical value

Tier 3:

Inverse operation: the operation that reverses the action

Solve: find a numerical value that satisfies an equation

Solve equations with brackets



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12 \qquad -12$

$6x = 18$

$\div 6 \qquad \div 6$

$\underline{x = 3}$

Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

$3x + 2 > 11$

Solve

$x \leftarrow -3 \leftarrow -2 \leftarrow 11$

$x > 3$

Inequalities with negatives

Method 1 Make x positive first

$2 - 3x > 17$

$+3x \qquad +3x$

$2 > 17 + 3x$

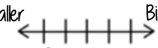
$-17 \qquad -17$

$-15 > 3x$

$\div 3 \qquad \div 3$

$\underline{-5 > x}$

x is true for any value smaller than -5



✓ CHECK IT!
$2 - 3(-6) = 20$
TRUE/ CORRECT

Method 2 Keep the negative x

$2 - 3x > 17$

$-2 \qquad -2$

$-3x > 15$

$\div -3 \qquad \div -3$

$\underline{x > -5}$

x is true for any value bigger than -5

This cannot be true…

$\underline{x < -5}$

When you multiply or divide x by a negative you need to reverse the inequality

Equations with unknown on both sides

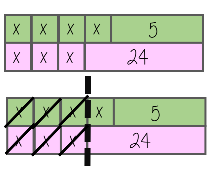
$4x + 5 = 3x + 24$

$-3x \qquad -3x$

$x + 5 = 24$

$-5 \qquad -5$

$\underline{x = 19}$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$5(x + 4) < 3(x + 2)$

$5x + 20 < 3x + 6$

$2x + 20 < 6$

$2x < -14$

$\underline{x < -7}$

Check it!

$5(-8 + 4) < 3(-8 + 2)$

$5(-4) < 3(-6)$

$-20 < -18$

✓ -20 IS smaller than -18

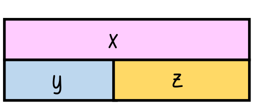
Formulae and Equations

Formulae – all expressed in symbols

Substitute in values

Equations – include numbers and can be solved

Rearranging Formulae (one step)



$x = y + z$

Rearrange to make y the subject.

$\underline{y = x - z}$

$y \longrightarrow +z \longrightarrow x$

$y \longleftarrow -z \longleftarrow x$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging…

Change the subject

Make XXX the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$4x - 3 = 9$

$+3 \qquad +3$

$4x = 12$

$\div 4 \qquad \div 4$

$\underline{x = 3}$

In a formula (make x the subject)

$xy - s = a$

$+s \qquad +s$

$xy = a + s$

$\div y \qquad \div y$

$x = \dfrac{a + s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g. Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \dfrac{4x + 9}{2}$

Gradient $= \dfrac{4}{2} = 2$

NUMBER…

Fractions

What do I need to be able to do?

- Add and subtract fractions
- Multiply and divide fractions
- Fraction of an amount

Keywords

Tier 2:

Whole: a positive number including zero without any decimal or fractional parts.

Dividend : the amount you want to divide up.

Divisor: the number that divides another number.

Quotient: the answer after we divide one number by another. e.g. dividend ÷ divisor = quotient

Tier 3:

Numerator : the number above the line on a fraction. The top number. Represents how many parts are taken

Denominator: the number below the line on a fraction. The number represent the total number of parts.

Unit Fraction: a fraction where the numerator is one and denominator a positive integer.

Non-unit Fraction: a fraction where the numerator is larger than one.

Fraction of a given amount

Find $\frac{2}{5}$ of £205

The bar represents the whole amount

£205

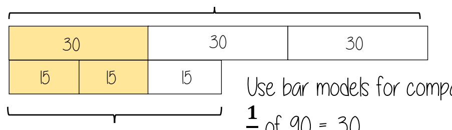
2 out of the 5 equal parts

2 x £41 = £82

£205 ÷ 5 = £41

Each part of the bar model represents £41.

90



45

Use bar models for comparisons

$\frac{1}{3}$ of 90 = 30

$\frac{2}{3}$ of 45 = 30

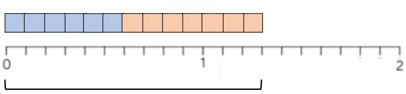
$\therefore \frac{1}{3}$ of $90 = \frac{2}{3}$ of 45

Add/Subtraction fractions (common multiples)

$$\frac{3}{5} + \frac{7}{10}$$

$$\frac{6}{10} + \frac{7}{10}$$

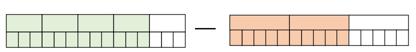
Addition/Subtraction needs a common denominator



$$\frac{13}{10}$$

Add/Subtraction any fractions

$$\frac{4}{5} - \frac{2}{3}$$



$$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

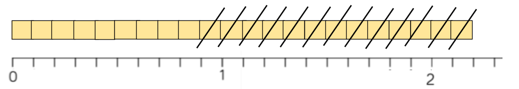
Use equivalent fractions to find a common multiple for both denominators

Add/Subtraction fractions (improper and mixed)

$$2\frac{1}{5} - 1\frac{3}{10}$$

$$2\frac{2}{10}$$

$$\frac{22}{10} - \frac{13}{10} = \frac{9}{10}$$



- Convert to an improper fraction
- Calculate with common denominator

Partitioning method

$$2\frac{1}{5} - 1\frac{3}{10} = 2\frac{2}{10} - 1\frac{3}{10} = 2\frac{2}{10} - 1 - \frac{3}{10} = 1\frac{2}{10} - \frac{3}{10} = \frac{9}{10}$$

Quick Multiplying and Cancelling down

$$\frac{\cancel{3}^{\,1}}{5} \times \frac{4}{\cancel{9}_{\,3}}$$

The 3 and the 9 have a common factor and can be simplified

Quick Solving

Multiply the numerators

Multiply the denominators

$$\frac{1 \times 4}{5 \times 3} = \frac{4}{15}$$

Multiplying fractions

Shade in 3 parts

Repeat it on this many rows

$$\frac{3}{4} \times \frac{2}{3}$$

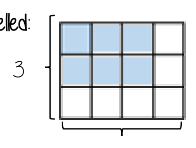
This many columns

This many rows

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

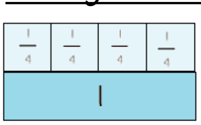
Parts shaded

Modelled:



Total number of parts in the diagram

Dividing an integer by a unit fraction



$$\underbrace{1 \div \frac{1}{4}}\ = 4$$

How many quarters are in 1?

"There are 4 quarters in 1 whole. Therefore, there are 20 quarters in 5 wholes"

$$5 \div \frac{1}{4} = 20$$

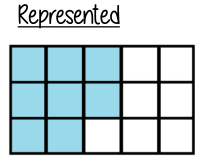
Dividing any fractions *Remember to use reciprocals*

$$\frac{2}{5} \div \frac{3}{4}$$

$$\frac{2}{5} \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

Represented



$$= \frac{8}{15}$$

PROPORTION... Rates

What do I need to be able to do?

- Speed, distance and time
- Distance-time graphs
- Flow problems and their graphs
- Rates of change and their units
- Convert compound units (E)

Keywords

Tier 2:

Convert: change

Mass: a measure of how much matter is in an object. Commonly measured by weight.

Origin: the coordinate (0, 0)

Volume: the amount of 3D space a shape takes up

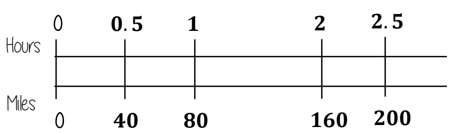
Substitute: putting numbers where letters are – replacing numbers into a formula

Speed, Distance, Time

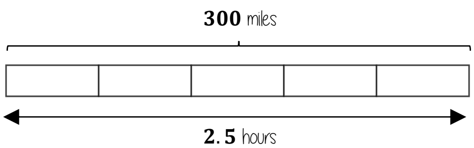
"per" for every
e.g. 80 miles per hour (mph)
Travel 80 miles every hour

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You can use a double number line to help you calculate distance



e.g. A boat travels at a constant speed for 2.5 hours. It travels 300 miles.



Bar models can help to calculate mph

Each part is half an hour
Each part is 60 miles

Speed, Distance, Time

minutes $\xrightarrow{\div\, 60}$ hours

hours $\xrightarrow{\times\, 60}$ minutes

Before calculations – make sure you are working in the same units as the speed

Learn or learn how to rearrange the formular for speed, distance and time

Substitute in the variables given

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

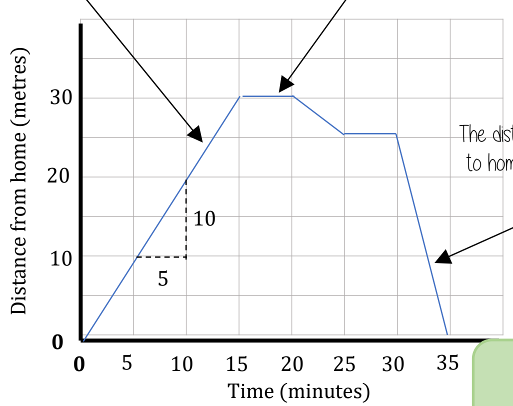
Distance – Time graphs

$$\text{Gradient} = \text{speed}$$

The steeper a gradient the faster the speed

$\frac{10}{5} = 2$ metres per min

Horizontal lines represent staying still



The distance coming closer to home shows the return journey

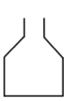
Units are important. Metres per minute

Converting compound units

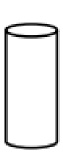
Converting compound units involves transforming both parts of a rate

3 metres per second	3 × 60 = 180
180 metres per minute	180 × 60 = 10 800
10 800 metres per hour	10 800 ÷ 1000 = 10.8
10.8 kilometres per hour	

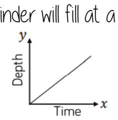
Flow problems & graphs



This will fill at a constant rate, then as the space decreases it will speed up and the neck of the bottle fill at a faster constant speed



The cylinder will fill at a constant speed



Units are important. Ensure any volume calculations are the same unit as the rate of flow

Rates of change & units

Common rates of change relationships

Revisit your conversions between units of length and capacity

Speed: miles **per** hour

Exchange rates: euros **per** pounds

Density: mass **per** volume

metres $\xrightarrow{\div\, 1000}$ kilometres

kilometres $\xrightarrow{\times\, 1000}$ metres

NUMBER…

Standard Form

What do I need to be able to do?

- Numbers in standard form
- Compare and order numbers in standard form
- Multiply and divide numbers in standard form
- Add and subtract numbers in standard form

Keywords

Tier 2:
Standard (index) Form: A system of writing very big or very small numbers
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: A value below zero.
Tier 3:
Commutative: an operation is commutative if changing the order does not change the result.

Positive powers of 10

1 billion – 1 000 000 000
$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

Standard form with numbers > 1

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Example
3.2×10^4
$= 3.2 \times 10 \times 10 \times 10 \times 10$
$= 32000$

Non-example
0.8×10^4
$5.3 \times 10^{0.7}$

Negative powers of 10

0.001
$1 \times \frac{1}{1000}$
1×10^{-3}

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
0	0	0	0	1

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers between 0 and 1

0.054
$= 5.4 \times 10^{-2}$

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	10^{-1}	10^{-2}	10^{-3}
0	0	5	4

A negative power does not mean a negative answer – it means a number closer to 0

Order numbers in standard form

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}

6.4×10^{-2}	2.4×10^2	3.3×10^0	1.3×10^{-1}
0.064	240	1	0.13

Look at the power first will the number be = > or < than 1

Use a place value grid to compare the numbers for ordering

Mental calculations

$6.4 \times 10^2 \times 1000$ Not in Standard Form
$= 6.4 \times 10^2 \times 10^3$ Use addition for indices rule
$= \underline{6.4 \times 10^5}$

$8 \times 10^5 \times 3$
$= 24 \times 10^5$ Not in Standard Form
$= 2.4 \times 10^1 \times 10^5$ Use addition for indices rule
$= \underline{2.4 \times 10^6}$

$(2 \times 10^3) \div 4$ Divide the values
$= (2 \div 4) \times 10^3$
$= \underline{0.5 \times 10^3}$

Remember the layout for standard form

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$6 \times 10^5 + 8 \times 10^5$

Method 1
$= 600000 + 800000$
$= 1400000$
$= \underline{1.4 \times 10^6}$

More robust method
Less room for misconceptions
Easier to do calculations with negative indices
Can use for different powers

Method 2
$= (6 + 8) \times 10^5$
$= 14 \times 10^5$ ← This is not the final answer
$= 1.4 \times 10^1 \times 10^5$
$= \underline{1.4 \times 10^6}$

Only works if the powers are the same

Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$ ← Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \times 10^5 \div 10^3$

$= \underline{5 \times 10^2}$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices
$a^m \times a^n = a^{m+n}$

Subtraction law for indices
$a^m \div a^n = a^{m-n}$

Using a calculator

$14 \times 10^5 \times 3.9 \times 10^3$

Use a calculator to work out this question to a suitable degree of accuracy

Input 14 and press [x10ˣ] Then press 5 (for the power)
Press [×]
Input 3.9 and press [x10ˣ] Then press 3 (for the power)
Press [=]

This gives you the solution

Click calculator for video tutorial

To put into standard form and a suitable degree of accuracy
Press [SHIFT] [SETUP] and then press 7 for sci mode
Choose a degree of accuracy so in most cases press 2

Answer: 5.5×10^8

NUMBER... Maths & Money

What do I need to be able to do?

- Understand a bank account
- Spending
- Ways to pay
- Ways to save
- Jobs and pay
- Investing
- Borrowing (buying a house)
- Running a house or a business
- Budgeting
- Borrowing (loans)
- Spending overseas
- Insurance

Keywords

Tier 2:

Credit: money being placed into a bank account
Debit: money that leaves a bank account
Balance: the amount of money in a bank account
Expense: a cost/outgoing
Deposit: an initial payment (often a way of securing an item you will later pay for)
Multiplier: a number you are multiplying by (Multiplier more than 1 = increasing, less than 1 = decreasing)
Per Annum: each year
Currency: the type of money a country uses.
Insurance: protection against financial loss
Policy: contract between an insurer and a policyholder
Cover: protection provided by an insurance policy against financial loss

Bills and Bank Statements

Bills – tell you the amount items cost and can show how much money you need to pay.

Some can include a total

Look for different units (Is it in pence or pounds)

Menu	Price
Milk	89p
Tea	£1.50

Bank Statements

Bank statement can have negative balances if the money spent is higher than the money coming into the account

Date	Description	Credit	Debit	Balance
19th Sept	Salary	£1500		£1500
19th Sept	Mortgage		£600	£900
25th Setp	Bday Money	£15		£915

Simple Interest

For each year of investment, the interest remains the same

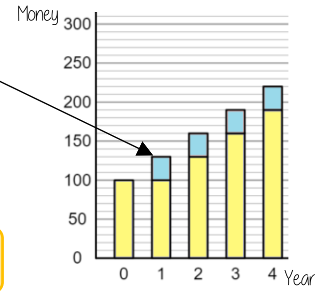
$$\frac{\text{Principal amount} \times \text{Interest Rate} \times \text{Years}}{100}$$

Principal amount is the amount invested in the account.

e.g Invest £100 at 30% simple interest for 4 years

$$\frac{100 \times 30 \times 4}{100} = £120$$

This account earned **£120** interest.
At the end of year 4 they have **£220**



Compound Interest

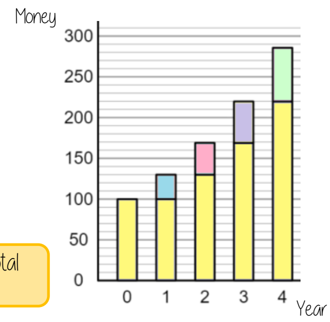
Interest is added to the current value of investment at the end of each year so the next year's interest is greater.

$$\text{Principal amount} \times \text{Multiplier}^{\text{Years}}$$

e.g Invest £100 at 30% compound interest for 4 years

$$100 \times 1.3^4 = £285.61$$

This account has **£285.61** in total at the end of the 4 years.



Value Added Tax (VAT)

VAT is payable to the government by a business. In the UK VAT is 20% and added to items that are bought.

Essential items such as food do not include VAT.

Wages and Taxes

Salaries fall into tax brackets – which means they pay this much each month from their salary.

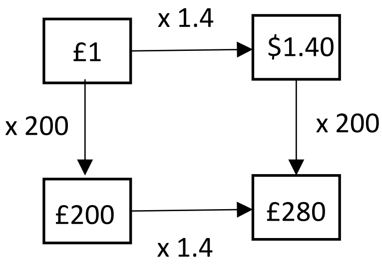
Taxable Income	Tax Rate
£12 501 to £50 000	20%
£50 001 to £150 000	40%
over £150 000	45%

Over time:

Time and a half – means 1.5 times their hourly rate

Double – 2 times their hourly rate

Exchange Rates



When making estimates it is also useful to use estimates to check if our solution is reasonable.

Use inverse operations to reverse the exchange process

Common Currencies		
United Kingdom	£	Pounds
United States of America	$	Dollars
Europe	€	Euros

Insurance

Insurance provides protection against financial loss.

Example: Car insurance policy

Annually	Excess	Additional products
£740.51	£500 voluntary £250 compulsory £750 total	✓ Personal accident cover Included as standard ✓ Courtesy car Included as standard ✗ Breakdown cover ✗ Motor legal protection Add annually £28.00
Monthly		
Monthly £67.88 × 11 Deposit £61.70		

ALGEBRA... Straight Line Graphs

What do I need to be able to do?

- Lines, parallel to the axes, y=x and y=-x
- Explore gradients
- Explore intercepts
- y=mx+c
- Rearrange equations to the form y=mx+c (E)
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs
- Graph inequalities (E)

Keywords

Tier 2:

Gradient: the steepness of a line

Intercept: where two lines cross. The y-intercept: where the line meets the y-axis.

Parallel: two lines that never meet with the same gradient.

Co-ordinate: a set of values that show an exact position on a graph.

Linear: linear graphs (straight line) – linear common difference by addition/ subtraction

Reciprocal: a pair of numbers that multiply together to give 1.

Perpendicular: two lines that meet at a right angle.

Tier 3:

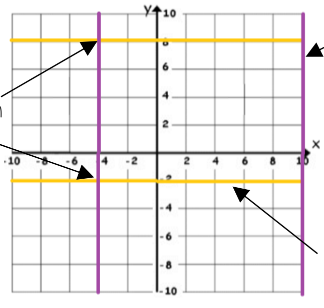
Asymptote: a straight line that a graph will never meet.

Lines parallel to the axes

All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Intersection points



Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

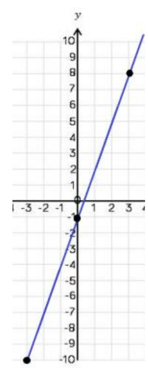
Plotting y = mx + c graphs

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	−3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

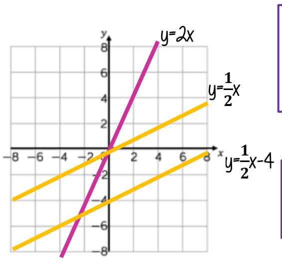
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

$y = mx + c$

The **coefficient** of x (the number in front of x) tells us the gradient of the line



The **greater** the gradient – the **steeper** the line

Parallel lines have the **same** gradient

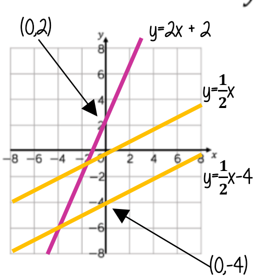
Positive gradients

Negative gradients

Compare Intercepts

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**



The coordinate of a y intercept will always be (0,c)

Lines with the **same** y-intercept cross in the **same** place

y = mx + c

The **coefficient** of x (the number in front of x) tells us the gradient of the line

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**.

The equation of a line can be rearranged. Eg:

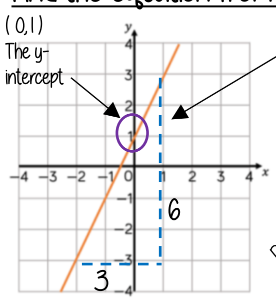
y = c + mx

c = y – mx

Identify which coefficient you are identifying or comparing

Find the equation from a graph

(0,1) The y-intercept



The Gradient $\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

The y-intercept shows the minimum charge. The gradient represents the price per mile

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs To represent direct proportion the graph must start at the origin.

When you have 0 pens this has 0 cost. The gradient shows the price per pen

A box of pens costs £2.30. Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

PROPORTION...

Ratio & proportion

What do I need to be able to do?

- Direct proportion
- Direct proportion and conversion graphs
- Inverse proportion
- Inverse proportion graphs (E)
- Ratio problems (whole or part given)
- Solve problems with ratio and algebra (E)

Keywords

Tier 2:

Proportion: a comparison between two numbers

Ratio: a ratio shows the relative size of two variables

Tier 3:

Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

Inverse proportion:: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

Direct Proportion

As one variable changes the other changes at the same rate.

This is a multiplicative change

4 cans of pop = £2.40

4 cans of pop = £2.40 (x 0.5)
2 cans of pop = £1.20 (x 0.5)

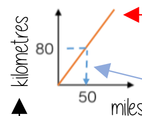
This multiplier is the same In the same way that this would be for ratio

4 cans of pop = £2.40 (x 3)
12 cans of pop = £7.20 (x 3)

Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

÷ 2   × 4

T	1	2	8
G	40	20	5

× 2   ÷ 4

Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts

Cost per item

Shop A
4 cans for £1.20
£1.20 ÷ 4
1 can is £0.30
Or 30p

Shop B
3 cans for 93p
£0.93 ÷ 3
1 can is £0.31
Or 31p

Shop **A** is the best value as it is 1p cheaper per can of pop

Cost per pound

Shop A
4 cans for £1.20
4 ÷ £1.20
£1 buys 3.333 cans of pop

3 cans for 93p
3 ÷ £0.93
£1 buys 3.23 cans of pop

Shop **A** is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound.

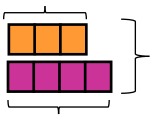
Best value is the most product for the lowest price per unit

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy
3 : 4



Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

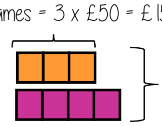
£350 ÷ 7 = £50

☐ = one part = £50

Put back into the question

James: Lucy
3 : 4 (x 50)
£150:£200

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

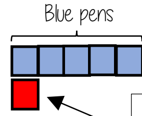
Finding a value given 1:n (or n:1)

Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red
5 : 1

☐ = one part = 10 pens

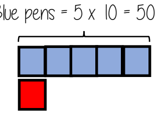


One unit = 10 pens

Put back into the question

Blue : Red
5 : 1 (x 10)
50: 10

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

GEOMETRY… Constructions & congruence

What do I need to be able to do?

- Draw and measure angles
- Construct and interpret scale drawings
- Construct triangles using ASA, SAS and SSS
- Construct an angle bisector
- Construct a perpendicular bisector
- Construct a perpendicular from or to a point
- Construct more complex polygons
- Identify congruent figures
- Congruent triangles

Keywords

Tier 2:

Locus: set of points with a common property

Equidistant: the same distance

Perpendicular: lines that meet at 90°

Arc: part of a curve

Bisector: a line that divides something into two equal parts

Congruent: the same shape and size

Tier 3:

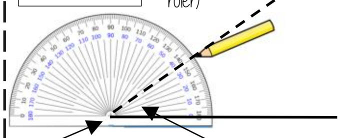
Protractor: piece of equipment used to measure and draw angles

Discorectangle: (a stadium) – a rectangle with semi circles at either end

Draw and measure angles

Draw a 35° angle

Make a mark at 35° with a pencil And join to the angle point (use a ruler)



The angle

Make sure the cross is at the end of the line (where you want the angle)

Scale drawings

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

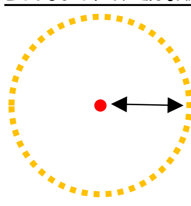
Image : Real life

1cm : 30cm

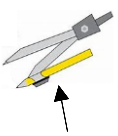
10cm : 300cm

× 10 × 10

Locus of a distance from a point

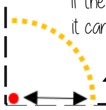


All points are equidistant (the same distance) from the fixed point in the middle.



Equipment needed

The radius is the distance from the fixed point

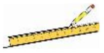


If the point is in the corner it can only make a quarter circle

Locus of a distance from a straight line

All points are equidistant (the same distance) from line

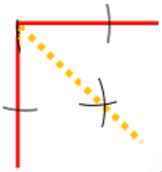
The ends of the line are fixed points



Equipment needed

The line is straight so a ruler is used for the straight lines parallel to your original line

Locus of a distance from two lines



Also an angle bisector

This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

Join the vertex to the intersection

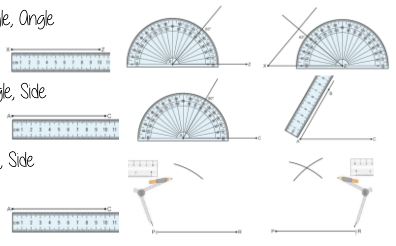
Constructing Triangles

Link to steps

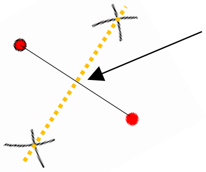
Side, Angle, Angle

Side, Angle, Side

Side, Side, Side



Locus equidistant from two points



Also a perpendicular bisector

Because if the points are joined, this new line intersects it at a 90°

Keep the compass the same size and draw two arcs from each point

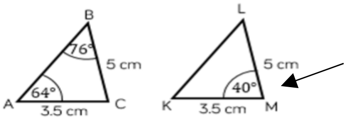
Join the intersections with a ruler.

All points on this line are equidistant from both points

Congruent figures

Congruent figures are identical in size and shape – they can be reflections or rotations of each other

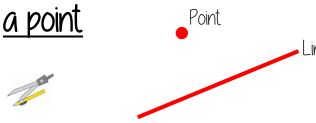
Congruent shapes are identical – all corresponding sides and angles are the same size



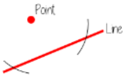
$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**

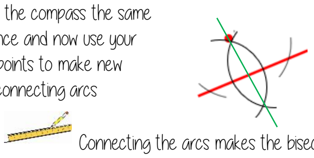
Construct a perpendicular from a point



Use a compass and draw an arc that cuts the line. Use the point to place the compass



Keep the compass the same distance and now use your new points to make new interconnecting arcs



Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

Congruent triangles

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

GEOMETRY...

Similarity

What do I need to be able to do?

- Recognise enlargement and similarity
- Work out unknown lengths and angles in similar shapes
- Solve problems with similar triangles (E)
- Ratio in right-angled triangles (E)

Keywords

Tier 2:

Scale Factor: the multiple describing how much a shape has been enlarged

Enlarge: to change the size of a shape (enlargement is not always making a shape bigger)

Corresponding: objects (or sides) that appear in the same place in two similar situations.

Image: the picture or visual representation

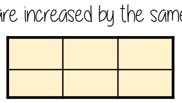
Tier 3:

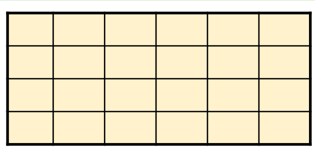
Similar Shapes: shapes of different sizes that have corresponding sides in equal proportion and identical corresponding angles.

Recognise enlargement & similarity

Shapes are similar if all pairs of corresponding sides are in the same ratio

These shapes are similar because all sides are increased by the same ratio

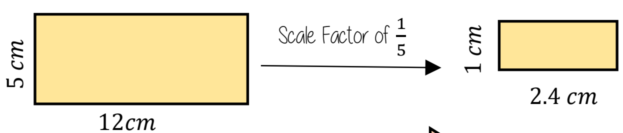


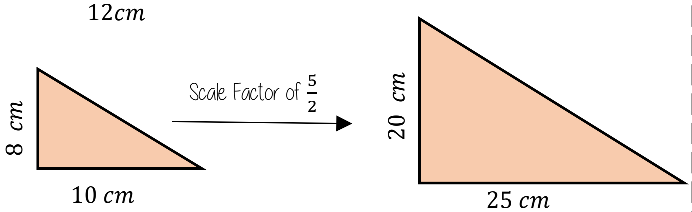


Enlargements are similar shapes with a ratio other than 1

Positive fractional scale factor

With a scale factor between 0 and 1 it makes the shape **smaller**

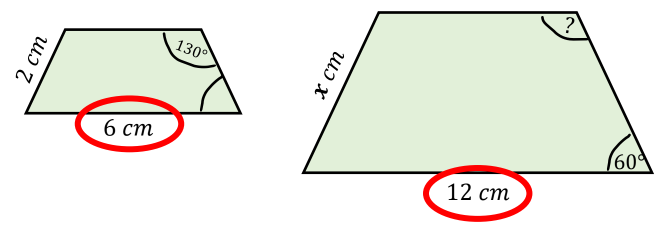




Calculations in similar shapes

Don't forget that properties of shapes don't change with enlargements or in similar shapes

The two trapezium are similar find the missing side and angle



Corresponding sides identify the scale factor $\frac{12}{6} = 2$ Scale Factor = **2**

Calculate the missing side Length (corresponding side) x scale factor

$2cm \times 2$

$x = 4cm$

Enlargement does not change angle size

Calculate the missing angle Corresponding angles remain the same

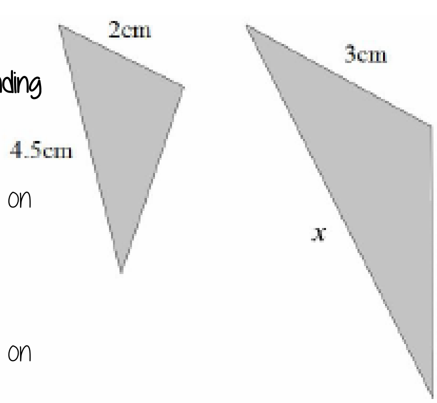
$130°$

Similar triangles

1. Find the scale factor.
2. Multiply or divide the corresponding side to find a missing length.

If you are finding a missing length on the larger shape you will need to multiply by the scale factor.

If you are finding a missing length on the smaller shape you will need to divide by the scale factor.



To show that two triangles are similar, show that:

1. The three sides are in the same proportion

2. Two sides are in the same proportion, and their included angle is the same

3. The three angles are equal

ALGEBRA…

Algebraic manipulation

What do I need to be able to do?

- Expand single brackets and simplify
- Factorise into a single bracket
- Expand double brackets
- Expand more complex double brackets
- Use identities
- Factorise quadratic expressions (E)
- Solve quadratic equations (E)
- Expand triple brackets (E)

Keywords

Tier 2:

Variable: a symbol for a number we don't know yet.

Expression: numbers, symbols and operators grouped together to show the value of something

Identity: An equation where both sides have variables that cause the same answer includes ≡

Substitute: replace one variable with a number or new variable.

sign)

Equivalent: something of equal value

Coefficient: a number used to multiply a variable

Tier 3:

Highest Common Factor (HCF): the biggest factor (or number that multiplies to give a term)

Algebraic constructs

Expression

A sentence with a minimum of two numbers and one maths operation

Term

A single number or variable

Identity

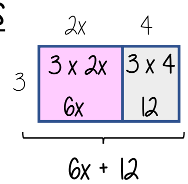
An equation where both sides have variables that cause the same answer includes ≡

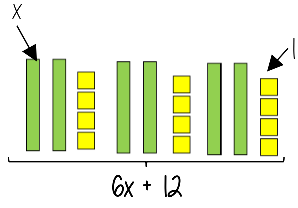
Expand single brackets

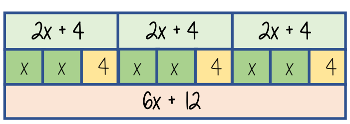
3 (2x + 4)

Different representations of

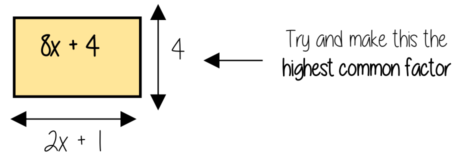
3(2x+4) = 6x + 12







Factorise into a single bracket



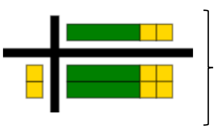
The two values **multiply** together (also the area) of the rectangle

8x + 4 ≡ 4 (2x + 1)

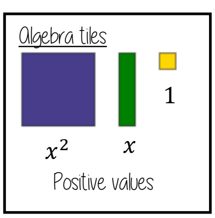
Note:
8x + 4 ≡ 2(4x + 2)
This is factorised but the HCF has not been used

Expand double brackets

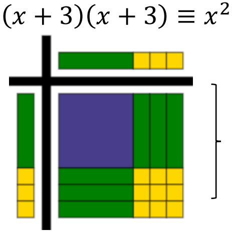
$2\ (x+2) \equiv 2x+4$



Algebra tiles can represent a binomial expansion
Has two terms



$(x+3)(x+3) \equiv x^2+6x+9$



This is a quadratic. It has four terms which simplified to three terms

The order of the brackets has no impact on the outcome.
e.g. $(x+3)(3+x)$

Factorising Quadratics

When a quadratic expression is in the form x^2+bx+c find the two numbers that **add to give** b and **multiply to give** c.

$x^2 + 7x + 10 = (x + 5)(x + 2)$

(because 5 and 2 add to give 7 and multiply to give 10)

Solving Quadratics

Factorise the quadratic in the usual way and then **solve** = 0.

Solve $x^2+3x-10=0$

Factorise: $(x+5)(x-2)=0$
$x=-5 \ or\ x=2$

Expanding triple brackets

Expand and simplify the following, $(x+1)(x+3)(x-2)$

Step 1 – Multiply out the first two brackets $(x+1)(x+3)$
$= x^2+3x+x+3$
$= x^2+4x+3$

Step 2 – Multiply the result (x^2+4x+3) by $(x-2)$

$(x^2+4x+3)(x-2)$
$= x^3+4x^2+3x-2x^2-8x-6$
$= x^3+2x^2-5x-6$

GEOMETRY...

Pythagoras' theorem

What do I need to be able to do?

- Solve equations with squares and square roots
- Identify the hypotenuse
- Determine whether a triangle is right-angled
- Pythagoras' theorem (find the hypotenuse)
- Pythagoras' theorem (find any side)
- Use Pythagoras' theorem on coordinate axes
- Proofs of Pythagoras' theorem (E)
- Pythagoras' theorem in 3-D shapes (E)

Keywords

Tier 2:

Opposite: the side opposite the angle of interest

Adjacent: the side next to the angle of interest

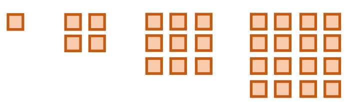
Tier 3:

Square number: the output of a number multiplied by itself

Square root: a value that can be multiplied by itself to give a square number

Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle.

Squares and square roots



This can also be written as 6^2

$\sqrt{\ }$ is the square root symbol

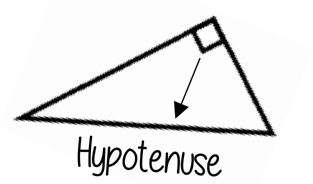
eg $\sqrt{64} = 8$

Because $8 \times 8 = 64$

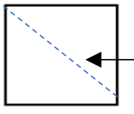
1×1	2×2	3×3	4×4	5×5	6×6	7×7	8×8	9×9	10×10
1	4	9	16	25	36	49	64	81	100

Square numbers

Identify the hypotenuse

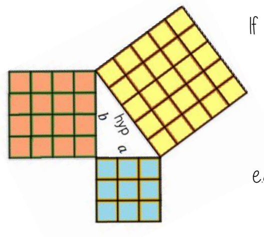


The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

Determine if a triangle is right-angled



a = 3 b = 4 c = 5

If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

$$a^2 + b^2 = \text{hypotenuse}^2$$

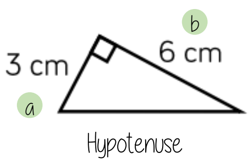
eg $a^2+b^2 = \textit{hypotenuse}^2$

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

Substituting the numbers into the theorem shows that this is a right-angled triangle

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1. Substitute in the values for a and b

$3^2+6^2 = \text{hypotenuse}^2$

$9 + 36 = \text{hypotenuse}^2$

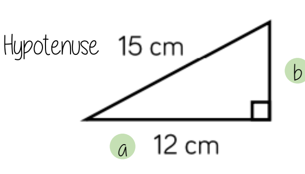
$45 = \text{hypotenuse}^2$

2. To find the hypotenuse square root the sum of the squares of the shorter sides.

$\sqrt{45} = \text{hypotenuse}$

$\mathbf{6.71\,cm} = \text{hypotenuse}$

Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$12^2+b^2 = 15^2$

1. Substitute in the values you are given

$144 + b^2 = 225$

$-144 \qquad -144$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

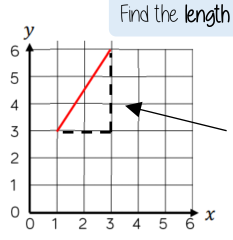
Square root to find the length of the side

$b^2 = 111$

$b = \sqrt{111} = 10.54\ cm$

Pythagoras' theorem on a coordinate axis

Find the **length** of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram.

The line segment is the **hypotenuse**

$$a^2 + b^2 = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle.

Be careful to check the scale on the axes

ALGEBRA...

Non-linear Graphs

What do I need to be able to do?

- Substitute into quadratic expressions
- Draw quadratic graphs
- Draw more complex quadratic graphs
- Interpret quadratic graphs
- Interpret reciprocal and exponential graphs
- Draw cubic graphs (E)
- Interpret cubic graphs (E)
- Interpret roots, intercepts and turning points (E)

Keywords

Tier 2:

Quadratic: a curved graph with the highest power being 2. Square power.

Reciprocal: a reciprocal is 1 divided by the number

Exponential: graphs where the variable is a power

Origin: the coordinate (0, 0)

Tier 3:

Cubic: a curved graph with the highest power being 3. Cubic power.

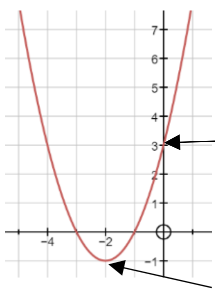
Parabola: a 'u' shaped curve that has mirror symmetry

Quadratic Graphs

$y = x^2 + 4x + 3$

If x^2 is the highest power in your equation then you have a quadratic graph.

It will have a parabola shape



Intersection with the y axis

Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

Coordinate pairs for plotting $(-3, 0)$

Plot all of the coordinate pairs and join the points with a curve (freehand)

Quadratic graphs are always symmetrical with the turning point in the middle

Interpret other graphs

Cubic Graphs

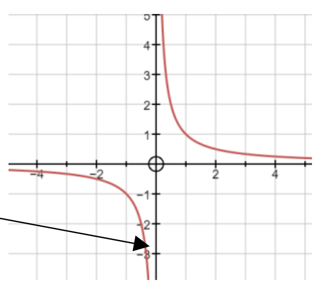
$y = x^3 + 2x^2 - 2x + 1$

If x^3 is the highest power in your equation then you have a cubic graph

Reciprocal Graphs

$y = \frac{1}{x}$

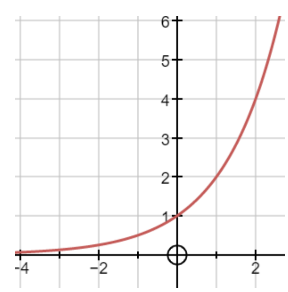
Reciprocal graphs never touch the y axis.
This is because x cannot be 0
This is an asymptote



Exponential Graphs

$y = 2^x$

Exponential graphs have a power of x



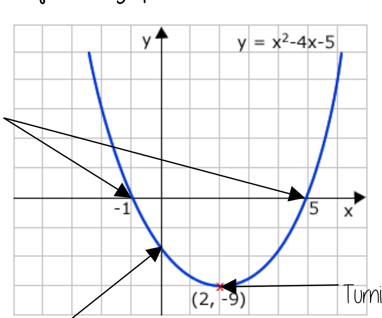
Identify and interpret roots, intercepts of quadratic functions graphically

A root is a **solution**.
The roots of a quadratic are the **x-intercepts of the quadratic graph**

A **turning point** is the point where a quadratic turns.
On a **positive parabola**, the turning point is called a **minimum**.

On a **negative parabola**, the turning point is called a **maximum**.

x- intercepts



Turning point

y- intercept

Substitution into expressions

4y ← 4 lots of 'y'

If y = 7 this means the expression is asking for 4 'lots of' 7

4 x 7 OR 7 + 7 + 7 + 7 OR 7 x 4 $\boxed{= 28}$

eg: y – 2
= 7 – 2 = 5

$\boxed{2(x + 3)}$ Put the expression into a function machine

INPUT → $\boxed{+ 3}$ → $\boxed{x\ 2}$ → OUTPUT

If x = 10   10 + 3 = 13.... 13 x 2 = 26

PROBABILITY...

Sets & Probability

What do I need to be able to do?

- Identify and represent sets
- Intersection of a set
- Union of a set
- Probability of a single event
- Use diagrams to work out probabilities
- Relative frequency
- Expected outcomes
- Independent events
- Probabilities from Venn diagrams

Keywords

Tier 2:

Probability: the chance that something will happen

Independent: an event that is not affected by any other events.

Chance: the likelihood of a particular outcome.

Event: the outcome of a probability – a set of possible outcomes.

Biased: a built in error that makes all values wrong by a certain amount.

Tier 3:

Relative Frequency: how often something happens divided by the outcomes

Identify and represent sets

The **universal set** has this symbol ξ – this means EVERYTHING in the Venn diagram is in this set

A set is a collection of things – you write sets inside curly brackets { }

ξ = {the numbers between 1 and 50 inclusive}

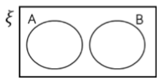
My sets can include every number between 1 and 50 including those numbers

A = {Square numbers}

A = {1, 4, 9, 16, 25, 36, 49}

All the numbers in set A are square number and between 1 and 50

Interpret and create Venn diagrams



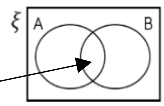
Mutually exclusive sets

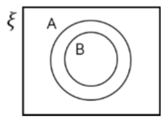
The two sets have nothing in common

No overlap

Union of sets

The two sets have some elements in common – they are placed in the intersection





Subset

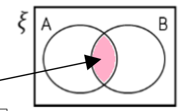
All of set B is also in Set A so the ellipse fits inside the set

The box

Around the outside of every Venn diagram will be a box. If an element is not part of any set it is placed outside an ellipse but inside the box

Intersection of sets

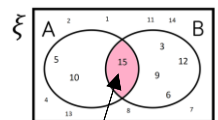
Elements in the intersection are in set A AND set B



The notation for this is $A \cap B$

ξ = {the numbers between 1 and 15 inclusive}

A = {Multiples of 5} B = {Multiples of 3}

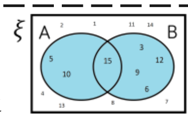


The element in $A \cap B$ is 15

In this example there is only one number that is both a multiple of 3 and a multiple of 5 between 1 and 15

Union of sets

Elements in the union could be in set A OR set B



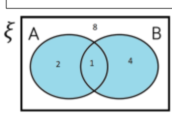
ξ = {the numbers between 1 and 15 inclusive}

A = {Multiples of 5} B = {Multiples of 3}

The notation for this is $A \cup B$

The elements in $A \cup B$ are 5, 10, 15, 3, 9, 6, 12

There are 7 elements that are either a multiple of 5 OR a multiple of 3 between 1 and 15



This Venn shows the **number of elements** in each set

Probability of a single event

Probability = $\frac{\text{number of times event happens}}{\text{total number of possible outcomes}}$

Probability notation P (event)

P(Blue) = $\frac{4}{10}$ = $\frac{2}{5}$

There are 4 blue sectors

There are 10 sectors overall

Probability can be a fraction, decimal or percentage value

$\frac{4}{10} = \frac{40}{100} = 0.40 = 40\%$

Probability is always a value between 0 and 1

Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

Colour	Frequency	Relative Frequency
Green	6	0.3
Yellow	12	0.6
Blue	2	0.1
	20	

Relative frequency can be used to find expected outcomes

e.g. Use the relative probability to find the expected outcome for green if there are 100 selections.

Relative frequency x Number of times $0.3 \times 100 = 30$

Expected outcomes

Expected outcomes are estimations.
It is a long term average rather than a prediction.

Dark	Milk	White
0.15	0.35	0.5

The sum of the probabilities is **1**

An experiment is carried out **400** times.
Show that dark chocolate is expected to be selected **60** times

$0.15 \times 400 = 60$

PROBABILITY...

Sets & Probability

Construct sample space diagrams

Sample space diagrams provide a systematic way to display outcomes from events

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

This is the set notation to list the outcomes S =

In between the { } are a; the possible outcomes

S = { 1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T}

Probability from sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

What is the probability that an outcome has an even number and a tails?

This is the set notation that represents the question P

P (Even number and Tails) = $\frac{3}{12}$

In between the ( ) is the event asked for

There are three even numbers with tails

There are twelve possible outcomes

Numerator: the event

Denominator: the total number of outcomes

Probability from two-way tables

	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

The total number of items

P (Girl walk to school) = $\frac{21}{100}$

The event

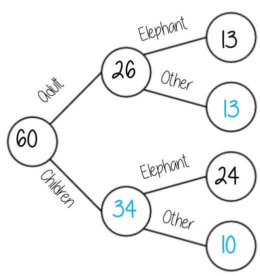
The total in the set

Product Rule

The number of items in event a X The number of items in event b

Probability from frequency trees

60 people visited the zoo one Saturday morning.
26 of them were adults.
13 of the adult's favourite animal was an elephant.
24 of the children's favourite animal was an elephant.



Frequency trees and two-way tables can show the same information

The total columns on two-way tables show the possible denominators

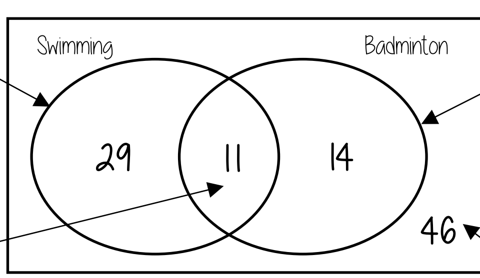
P(adult) = $\frac{26}{60}$

P(Child with favourite animal as elephant) = $\frac{13}{37}$

Probability from Venn diagrams

100 students were questioned if they played badminton or went to swimming club.
40 went swimming, 25 went to badminton and 11 went to both.

This whole curve includes everyone that went swimming. Because 11 did both we calculate **just** swimming by 40- 11



This whole curve includes everyone that went to badminton. Because 11 did both we calculate **just** badminton by 25 - 11

P (Just swimming) = $\frac{29}{100}$

The intersection represents both Swimming AND badminton

The number outside represents those that did **neither** badminton or swimming

100 – 29 – 11 – 14

GEOMETRY... Transformations

What do I need to be able to do?

- Enlargement (positive scale factor)
- Enlargement from a point (positive scale factor)
- Enlargement (fractional scale factor)
- Enlargement (negative scale factor) (E)
- Describe an enlargement
- Rotation about a point
- Describe a rotation
- Translation
- Describe a translation
- Reflection
- Find the result of a series of transformations (E)

Keywords

Tier 2:

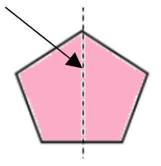
Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Similar: when one shape can become another with a reflection, rotation, enlargement or translation.
Reflect: mapping of one object from one position to another of equal distance from a given line.
Rotate: a rotation is a circular movement.
Vertex: a point where two or more-line segments meet.
Perpendicular: lines that cross at 90°
Symmetry: when two or more parts are identical after a transformation.
Regular: a regular shape has angles and sides of equal lengths.
Invariant: a point that does not move after a transformation.

Tier 3:

Scale Factor: the multiplier of enlargement
Centre of enlargement: the point the shape is enlarged from

Lines of symmetry

Mirror line (line of reflection)

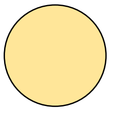


Rhombus
two lines of symmetry

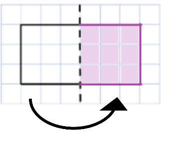
Parallelogram
No lines of symmetry

Shapes can have more than one line of symmetry….
This regular polygon (a regular pentagon has 5 lines of symmetry)

a circle has an infinite amount of lines of symmetry

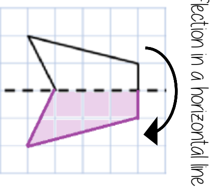


Reflect horizontally/ vertically (1)



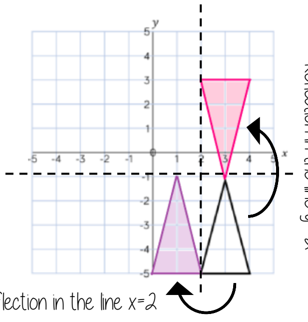
Note: a reflection doubles the area of the original shape

Reflection in a vertical line



Reflection in a horizontal line

Reflection on an axis grid

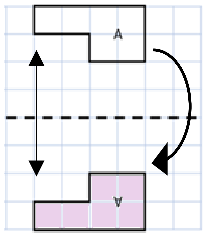


Reflection in the line y=-2

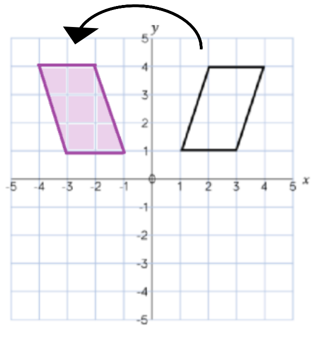
Reflection in the line x=2

Reflect horizontally/ vertically (2)

All points need to be the same distance away from the line of reflection



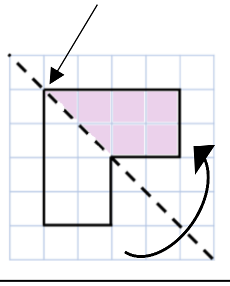
Reflection in the line y axis – this is also a reflection in the line x=0



Lines parallel to the x and y axis
REMEMBER
Lines parallel to the x-axis are y = ____
Lines parallel to the y-axis are x = ____

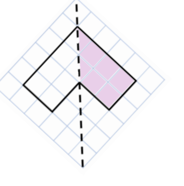
Reflect Diagonally (1)

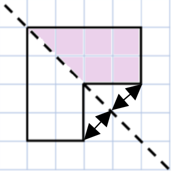
Points on the mirror line don't change position



Fold along the line of symmetry to check the direction of the reflection

Turn your image
If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

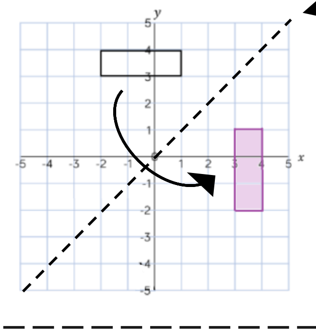




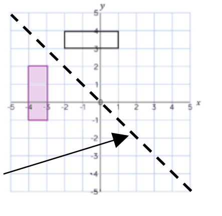
Perpendicular lines to and from the mirror line can help you to plot diagonal reflections

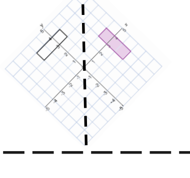
Reflect Diagonally (2)

This is the line **y = x** (every y coordinate is the same as the x coordinate along this line)



This is the line **y = - x**
The x and y coordinate have the same value but opposite sign



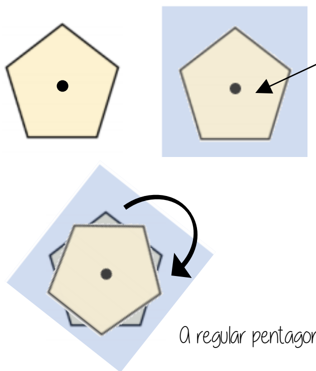


Turn your image
If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

GEOMETRY... Transformations

Rotational Symmetry

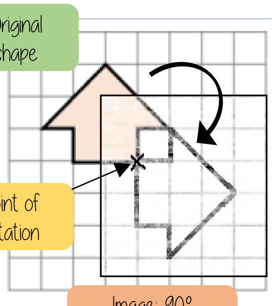
Tracing paper helps check rotational symmetry



1. Trace your shape (mark the centre point)
2. Rotate your tracing paper on top of the original through 360°
3. Count the times it fits back into itself

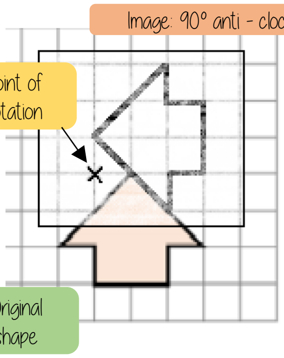
a regular pentagon has rotational symmetry of order 5

Rotate from a point (in a shape)



1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

Rotate from a point (outside a shape)



1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

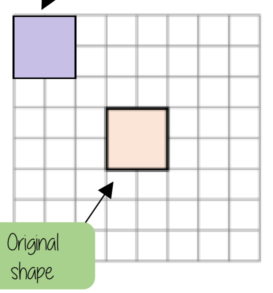
Translation and vector notation

Vector Notation → $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

How far left or right to move
Negative value (left)
Positive value (right)

How far up or down to move
Negative value (down)
Positive value (up)

Translation $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$

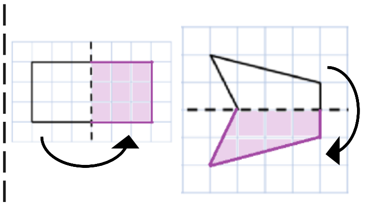


Every vertex has been translated by the same amount

$\begin{pmatrix} -3 \\ 3 \end{pmatrix}$

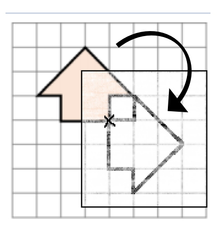
The image has been moved 3 squares to the left and 3 squares up

Compare rotations and reflections



Reflections are a mirror image of the original shape.

Information needed to perform a reflection:
- Line of reflection (Mirror line)



Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation:
- Point of rotation
- Direction of rotation
- Degrees of rotation

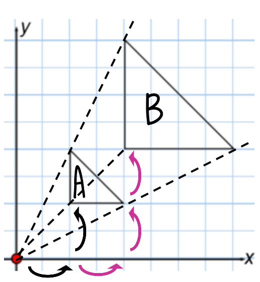
Positive scale factors

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

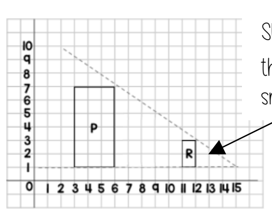
The distance from the point enlarges by 2



Fractional scale factors

Fractions less than 1 make a shape SMALLER

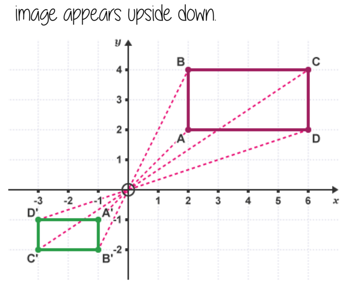
R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15,1)



SF: $\frac{1}{3}$ - R is three times smaller than P

Negative scale factors(H)

An enlargement with a negative scale factor produces an image on the other side of the centre of enlargement. The image appears upside down.



The rectangle ABCD has been enlarged by a scale factor of $-\frac{1}{2}$

ALGEBRA... Simultaneous Equations

What do I need to be able to do?

- Use one value to find another
- Introduction to simultaneous equations
- Solve simultaneous equations using graphs
- Solve simultaneous equations (no adjustments)
- Manipulating equations
- Solve simultaneous equations (adjust one)
- Solve simultaneous equations (adjust both) (E)
- Solve simultaneous equations by substitution (E)

Keywords

Tier 2:

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet.

Equation: an equation says that two things are equal – it will have an equals sign =

Substitute: replace a variable with a numerical value

Eliminate: to remove

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Coordinate: a set of values that show an exact position.

Intersection: the point two lines cross or meet.

Tier 3:

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

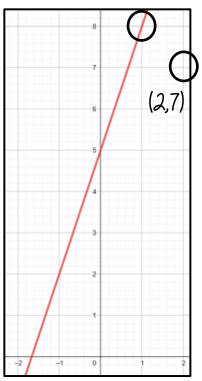
Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8

$y = 3x + 5$

$8 = 3(1) + 5$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5



Is (2,7) on the same line?

$7 \neq 3(2) + 5$

No 7 does NOT equal 6+5

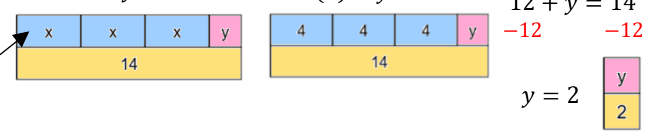
Substituting known variables

A line has the equation 3x + y = 14

Two different variables, two solutions

Stephanie knows the point x = 4 lies on that line. Find the value for y

$3x + y = 14$ $\quad$ $3(4) + y = 14$



$x = 4$

$12 + y = 14$

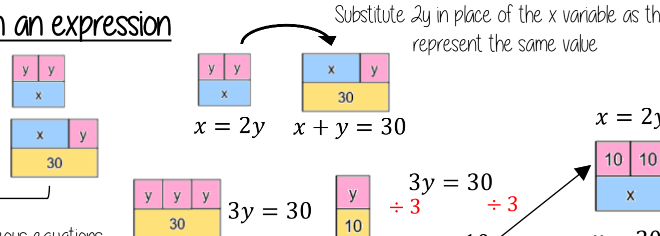
$-12 \quad -12$

$y = 2$

Substituting in an expression

x = 2y

x + y = 30



Pair of simultaneous equations (two representations)

Substitute 2y in place of the x variable as they represent the same value

$x = 2y$ $\quad$ $x + y = 30$

$3y = 30$

$3y = 30$

$\div 3 \quad \div 3$

$y = 10$

$x = 2y$

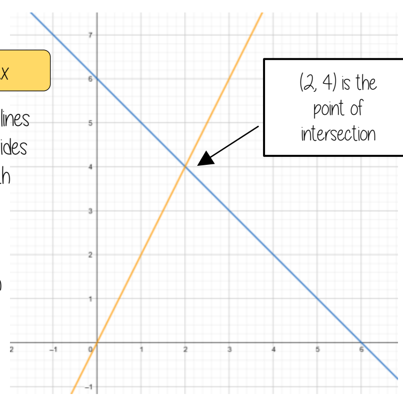
$x = 20$

Solve graphically

x + y = 6

y = 2x

Linear equations are straight lines. The point of intersection provides the x and y solution for both equations

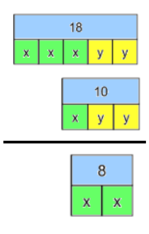


(2, 4) is the point of intersection

The solution that satisfies both equations is

$x = 2$ and $y = 4$

Solve by subtraction



$3x + 2y = 18$

$-\quad x + 2y = 10$

$2x = 8$

$\div 2 \quad \div 2$

$x = 4$

$x + 2y = 10$

$(4) + 2y = 10$

$-4 \quad -4$

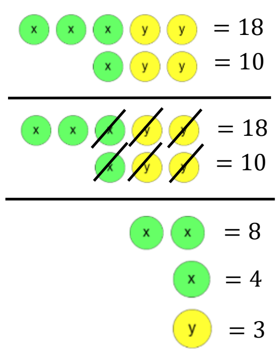
$2y = 6$

$\div 2 \quad \div 2$

$y = 3$

$x = 4$

$y = 3$



Solve by addition

$3x + 2y = 16$

$+\ 6x - 2y = 2$

$9x = 18$

$\div 9 \quad \div 9$

$x = 2$

$3x + 2y = 16$

$3(2) + 2(y) = 16$

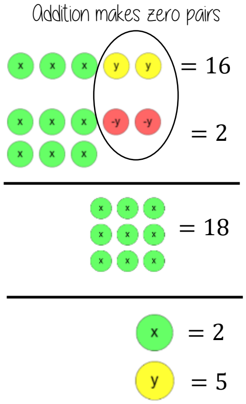
$6 + 2y = 16$

$-6 \quad -6$

$2y = 10$

$y = 5$

Addition makes zero pairs

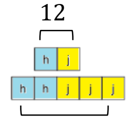


Solve by adjusting one

$h + j = 12$

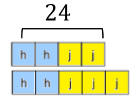
$2h + 2j = 29$

No equivalent values



$2h + 2j = 24$

$2h + 2j = 29$

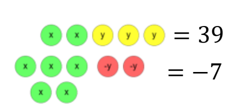


By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

Solve by adjusting both

$2x + 3y = 39$

$5x - 2y = -7$

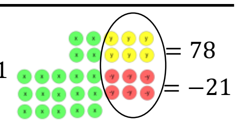


Use LCM to make equivalent x OR y values. Because of the negative values using zero pairs and y values is chosen choice

$4x + 6y = 78$

$15x - 6y = -21$

Now solve by addition



Addition makes zero pairs

GEOMETRY... Trigonometry

What do I need to be able to do?

- Identify hypotenuse, opposite and adjacent sides
- Use the tangent ratio to find unknown side lengths
- Use sine and cosine ratios to find unknown side lengths
- Use sine, cosine and tangent ratios to find unknown angles
- Choose the right method
- Key angles in right-angled triangles (E)
- Trigonometry in 3-D shapes (E)

Keywords

Tier 2:

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Constant: a value that remains the same

Inverse: function that has the opposite effect.

Tier 3:

Scale Factor: the multiplier of enlargement

Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.

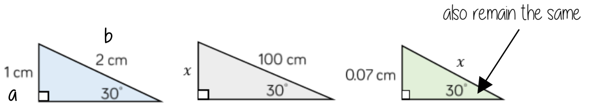
Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.

Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side

Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same



a : b
1 : 2

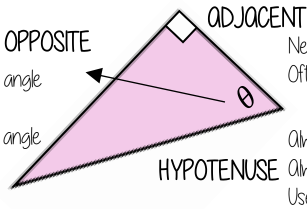
$\frac{a}{b} = \frac{1}{2}$

a : b
x : 100
50 : 100

a : b
0.07 : x
0.07 : 0.14

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE: Always opposite an acute angle. Useful to label second. Position depend upon the angle in use for the question

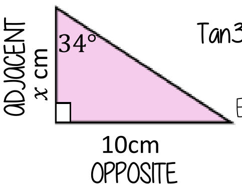
ADJACENT: Next to the angle in question. Often labelled last

HYPOTENUSE: Always the longest side. Always opposite the right angle. Useful to label this first

Tangent ratio: side lengths

$\text{Tan}\,\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



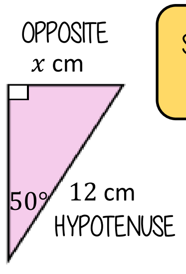
$\text{Tan}34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times \text{Tan}34 = 10$

$x = \frac{10}{\text{Tan}34} = 14.8\text{cm}$

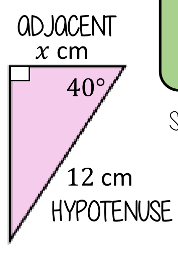
Sin and Cos ratio: side lengths



$\text{Sin}\,\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE

The Sin(x) ratio is the same as the Cos(90-x) ratio



$\text{Cos}\,\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

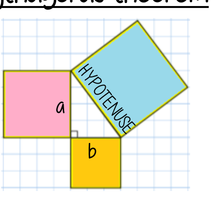
Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem

50°

$\text{Hypotenuse}^2 = a^2 + b^2$



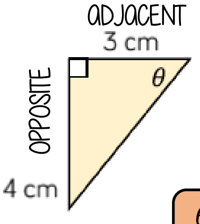
This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras

- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\theta = \text{Tan}^{-1}\,\frac{\text{opposite side}}{\text{adjacent side}}$

$\theta = \text{Sin}^{-1}\,\frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = \text{Cos}^{-1}\,\frac{\text{adjacent side}}{\text{hypotenuse side}}$

$\text{Tan}\,\theta = \frac{3}{4}$

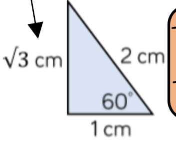
$\theta = \text{Tan}^{-1}\,\frac{3}{4}$

$\theta = 36.9°$

Key angles

Because trig ratios remain the same for similar shapes you can generalise from the following statements

This side could be calculated using Pythagoras



$\text{Tan}30 = \frac{1}{\sqrt{3}}$

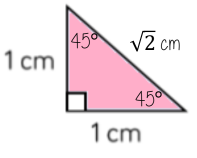
$\text{Tan}60 = \sqrt{3}$

$\text{Cos}30 = \frac{\sqrt{3}}{2}$

$\text{Cos}60 = \frac{1}{2}$

$\text{Sin}30 = \frac{1}{2}$

$\text{Sin}60 = \frac{\sqrt{3}}{2}$



$\text{Tan}45 = 1$

$\text{Cos}45 = \frac{1}{\sqrt{2}}$

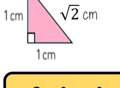
$\text{Sin}45 = \frac{1}{\sqrt{2}}$

Key angles 0° and 90°

$\text{Tan}0 = 0$

~~Tan90~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



$\text{Sin}0 = 0$

$\text{Sin}90 = 1$

$\text{Cos}0 = 1$

$\text{Cos}90 = 0$